ADDENDUM D6J:

AQUIFER-TEST THEORY

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D6J.1 AQUIFER-TEST THEORY

In order to determine fluid movement through an aquifer, a number of characteristics must be taken into account. Transmissivity is defined as the ability of an aquifer to transmit water and is usually expressed as gallons per day per foot (gal/day/ft). Transmissivity, expressed in these units, is the rate at which water flows through a unit width of an aquifer under a unit hydraulic gradient[2]. Transmissivity must be adjusted by the actual aquifer width and hydraulic gradient to determine actual aquifer flow rates.

Horizontal hydraulic conductivity (permeability) of the aquifer is the transmissivity divided by the aquifer thickness. Permeability is the main parameter that governs the velocity of groundwater movement. Hydraulic gradient and effective porosity are also needed with permeability to determine the velocity.

The storage coefficient, as defined by Theis, is the volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head. The storage coefficient is dimensionless. An unconfined aquifer derives water from compression of the aquifer and expansion of the water.

D6J.1.1 THEIS EQUATION

Theis, in 1935, introduced his equation to determine drawdowns in a non-leaky, confined aquifer. The following is a general definition of the Theis equation:

$$T = \frac{114.6Q\,W(u)}{s}$$

$$u = \frac{2693r^2S}{Tt}$$

where: s = drawdown, in feet
Q = discharge, in gallons per minute (gpm)
W(u) = well function, the integral from u to infinity of $(e^{-u})/u$ du
T = Transmissivity
u = well function variable
r = observation well radius from pumping well, in feet
S = storage coefficient
and t = time since pumping started, in minutes

Pump test data are analyzed by matching the log-log plot of drawdown versus time to Theis' type curve [W(u) vs. 1/u] and applying the above equations to the match[3].
The value of the integral expression for W(u) is given by the following series:

$$W(u) = -0.577216 - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} \ldots$$

where all terms are as previously defined.

D6J.1.1.1 STRAIGHT LINE EQUATION

Jacob developed a simplified form of Theis' drawdown equation by truncating the well function series after the first two terms. Assuming the truncation, the following equations were developed to analyze drawdown versus time data on semi-log plots and are called the straight-line or Jacob equation:

$$T = 264\ Q[\log (t_2/t_1)]/(s_2 - s_1)$$
$$T = 264\ Q/\Delta s$$
$$S = T\ t_o/4800r^2$$

s_1 = drawdown, in feet, at time since pumping started, t_1, in minutes

s_2 = drawdown, in feet, at time since pumping started, t_2, in minutes

and $t_2 > t_1$

Δs = change in drawdown over one log cycle of time on a semi-log Plot, in feet

S = storage coefficient

t_o = straight-line intercept of zero drawdown, in minutes

r = radius of well, in feet

A straight line is fitted to the semi-log plot of drawdown versus time (log scale) to obtain transmissivity. Jacob suggested the u values less than 0.01 are needed before his straight-line method is useful. However, a plot of W(u) versus 1/u on semi-log paper indicates that this method should be applicable for values of u as large as 0.1. Kruseman and de Rider (1991) suggest the use of a u of less than 0.1 to meet the Jacob condition[4].

D6J.1.1.2 THEIS RECOVERY EQUATION

Theis' equation can be modified to handle recharge of a well or multiple pumping periods by summation of the well functions. The following equation is the solution of Theis' equation for one pumping and recharge cycle (Recovery equation) of a non-leaky confined aquifer using a log-log match format:

$$T = 114.6\ Q\ [W(u) - W(u')]s'$$
$$u^1 = 2693\ r^2\ S/Tt'$$
$$T = 114.6\ Q\ [W(u) - W(u) + W(u')]\ sr$$
$$= 114.6\ Q\ W(u')/sr$$
$$s_r = s - s'$$

where: s_r = recovery, in feet

s' = residual drawdown (static water level – water level @ t'), in feet

$W(u')$ = recovery well function
u' = recovery well function variable
t' = time since pumping stopped, in minutes

The recovery data sets are analyzed by matching the log-log plot of the recovery versus time since pumping stopped to Theis' type curve. The type curve variables are $W(u')$ and $1/u'$ for the recovery match. The recovery is computed by estimating the drawdown which would have occurred if pumping had continued, and subtracting this predicted drawdown from the residual drawdown. For example, the recovery at 100 minutes after pumping has stopped is computed by estimating the drawdown had the pumping continued uninterrupted, and subtracting the estimated drawdown from the residual drawdown. The straight-line fit of the drawdown is normally extended to obtain these estimates of drawdown.

The well functions of the residual-drawdown form of Theis' equation were approximated by using the first two terms in the well function series. The following equations present the semi-log form of the Theis recovery equation:

$$T = 264\ Q\ [\log (t/t')]/s'$$

or

$$T = 264\ Q/\Delta s'$$

where: t = time since pumping started, in minutes
t' = time since pumping stopped, in minutes
s' = residual drawdown, in feet
and $\Delta s'$ = change in residual drawdown over one log cycle of t/t' on a semi-log plot, in feet

Therefore, when residual drawdown is plotted on an arithmetic scale versus t/t' on a logarithmic scale, the above equation can be used for the straight line fit[5]. The Theis equations were used to analyze data from the PW1 test.

D6J.1.1.3 MULTI-WELL THEIS EQUATION

The Theis equation can be modified to predict drawdown from more than one pumping well. Stallman[6] used the well function summation theory to develop type curves for a variable discharge pump test. HYDRO has used the well summation theory to analyze numerous pump tests with more than one pumping well. The sum of the $W(u)$ times Q values that are plotted versus $1/u_1$, on log-log paper to create the type curves. The following equations are for two pumping wells that start pumping at the same time:

$$T = 114.6/s\ [W(u_1)Q_1 + W(u_2)Q_2]$$
$$W(u1) = -0.577216 - \ln u_1 + u_1 - u_1^2/2.2! + u_1^3/3.3! - u_1^4/4.4!$$
$$u_1 = 1.87 r_1^2 S/Tt$$

where: parameters are the same as before, plus:

u_1 = well function variable for pumping well 1
u_2 = well function variable for pumping well 2
Q_1 = discharge for pumping well 1, in gpm
Q_2 = discharge for pumping well 2, in gpm

r_1 = observation well radius from pumping well 1, in ft.
r_2 = observation well radius from pumping well 2, in ft.

The summation of the product of the well functions and their corresponding discharge

$$\left(\sum_{i=1} [W(ui)Qi]\right)$$

are plotted against the inverse of the well function variable for the first pumping well (1/u1). If the discharge for each pumping well is the same, Q can be extracted from the summation term and taken as constant.

$$\frac{114.6Q}{s}\sum_{i=1}^{2} W(u_i)$$

The log-log plot of drawdown versus time for each observation well is then matched to its individual type curve to obtain the aquifer properties.

D6J.1.1.4 MULTI-WELL STRAIGHT-LINE EQUATION

The above Theis equation for two pumping wells can be modified using Jacob's approximation (see pp. 98-100 of Ferris, 1962) to obtain a straight-line (semi-log plot) for the drawdown data from two pumping wells. The u value of all wells must meet the straight-line assumptions before the straight-line method is applicable for the combined drawdown. An adequate straight-line will be developed at some observation wells early in the test when only the close pumping well has an influence. As with the single-well tests, u values should be less than 0.1 before the use of the straight-line method. The following is the derivation of the straight-line equation that is equivalent to Jacob's equation for two pumping wells at the same pumping rate:

$$s = \left(\frac{264Q}{T}\right)\left[\log\left(\frac{0.3Tt}{r_1^2 S}\right) + \log\left(\frac{0.3Tt_a}{r_2^2 S}\right)\right]$$

For drawdown at times of t_a and t_b:

$$s_b - s_a = \left(\frac{264Q}{T}\right)\left[\log\left(\frac{0.3Tt_b}{r_1^2S}\right) + \log\left(\frac{0.3Tt_b}{r_2^2S}\right) - \log\left(\frac{0.3Tt_a}{r_1^2S}\right) - \log\left(\frac{0.3Tt_a}{r_2^2S}\right)\right]$$

after multiplication and simplification of the log terms:

$$s_b - s_a = \frac{264Q}{T}\left[\log\left(\frac{t_b^2}{t_a^2}\right)\right]$$

$$s_b - s_a = \frac{264Q}{T}\left[\log\left(\frac{t_b}{t_a}\right)\right]$$

$$T = \frac{264Q(2)}{\Delta s} \text{ for } \Delta s = s_b - s_a \text{ for one log cycle}$$

The straight line equation is the same as the Jacob equation except the numerator is multiplied by two. The following is our derivation of the storage coefficient equation for two pumping wells starting at the same time:

$$s = o = \left(\frac{Q}{4\pi T}\right)\left[\ln\left(\frac{2.25Tt_o}{r_a^2S}\right) + \ln\left(\frac{2.25Tt_o}{r_b^2S}\right)\right]$$

$$o = \ln\left[\left(\frac{2.25Tt_o}{r_a^2S}\right)\left(\frac{2.25Tt_o}{r_b^2S}\right)\right]$$

$$1 = \left(\frac{2.25Tt_o}{S}\right)^2\left(\frac{1}{r_a^2r_b^2}\right)$$

$$S^2 = \left(\frac{2.25Tt_o}{(r_ar_b)}\right)^2$$

$$S = \left(\frac{2.25Tt_o}{(r_ar_b)}\right)$$

or in the usual USGS units

$$S = \frac{(0.3Tt_o)}{(r_ar_b)}$$

where: parameters are the same as before, plus:

s_b = drawdown, in feet, at time since pumping started, t_b, in days
s_a = drawdown, in feet, at time since pumping started, t_a, in days
t_o = time when drawdown equals zero (extension of straight-line fit to s = 0), in days

The drawdown data for an observation well are plotted on semi-log paper against times since the two wells began pumping. The slope of the straight-line fit is used with the discharge to compute the transmissivity, and the intercept of the straight line is used with the well radii to compute the storage coefficient. The multi-well equations were used to analyze data from the test in which both B12 and B14 were pumped.

D6J.1.2 HANTUSH'S MODIFIED METHOD

Hantush (1960) presents a modification of the theory of leaky confined aquifers which had previously been described by Hantush and Jacob (1955). The modification took into account the storage of water in the semipervious confining bed. Equations developed are as follows:

$$T = \frac{114.6Q}{s} H(u, BETA)$$

where: H(u, BETA) = the integral from u to infinity of (e^-y)/y [complementary error of the function of (BETA/Square Root U) / Square Root (y(y-u))]dy

u = [(2693)r^2(S)]/Tt

And BETA = r/4b Square Root (K' Ss' / K Ss)

The main parameters are as follows:

T = transmissivity, gal/day/ft.
Q = discharge, gpm
s = drawdown, ft.
y = variable of integration
r = radius, ft.
S = storage coefficient
t = time, min.
b = aquifer thickness, ft.
K = aquifer permeability, ft/day
K' = confining layer permeability, ft/day
Ss = aquifer specific storage, 1/ft.
and Ss' = confining layer specific storage, 1/ft.

This form of the beta equation assumes all leakage is coming from only one of the two confining layers. Hantush (1961) presented tabulations of H(u, BETA) for varying values of u and BETA, and subsequently, a family of type curves showing H(u, BETA) vs. 1/u has been developed. Main aquifer properties can be determined by matching plots of observed drawdown versus time data to one of Hantush's type curves and using the equations presented above. The specific storage of the confining layer can be determined from laboratory measurements of the coefficient of compressibility and void ratio on a core of the aquitard (see Section F.4). The specific storage of the aquifer if the laboratory measurements are not available.

D6J.1.3 NEUMAN-WITHERSPOON METHOD

A method for determining aquitard vertical permeability has been described by Neuman and Witherspoon (1971) and Neuman and Witherspoon (1972). In this technique, referred to as the Ratio Method, the ratio of drawdown in the aquitard to the drawdown in the pumped aquifer at the same time distance is related to a dimensionless time parameter, t'D:

t'D = K't / Ss z^2

where: K' = aquitard vertical permeability
t = time for which drawdown ration was determined
Ss' = specific storage of the aquitard
= K' / ALPHA'
ALPHA = aquitard diffusivity,
and z = vertical distance from the center of the screened section of the well completed in the aquitard to the aquifer.

t'D is determined graphically. Therefore, aquitard diffusivity (ALPHA') can be calculated from ALPHA' = K' / Ss' = T'D Z^2 / t.

In order to determine aquitard specific storage, Ss', must be ascertained.

Ss' = avWw / (1 + e)

where: av = coefficient of compressibility
Ww = weight of water,
and e = void ratio

The values of av and e must be determined on samples of the aquitard in the laboratory or Ss' may be estimated based on published reports on similar sediments.

D6J.1.4 DIRECTIONAL TRANSMISSIVITY

Directional transmissivity of the aquifer was quantified using a method described by Papadopulos (1965). Papadopulos derived an equation for the drawdown distribution around a well discharging at a constant rate from an infinite horizontal anisotropic aquifer. Aquifer-test data from a minimum of three observation wells are analyzed to obtain principal transmissivities and the orientation of the principal axes.

The equations derived by Papadopulos for use in a type-curve matching technique are as follows:

$$s = \frac{114.6Q\ W(Uxy)}{\left[(Txx)(Tyy) - Txy^2\right]^{1/2}}$$

and

$$Uxy = \frac{(1.87S)}{(t)} \frac{\left[(Txx)(y^2) + (Tyy)(x^2) - (2Txy)(x)(y)\right]}{\left[(Txx)(Tyy) - Txy^2\right]}$$

where s = drawdown, in feet
Q = discharge, in gpm
W(Uxy) = well function
Txx, Tyy & Txy = transmissivity components, in gal/day/ft
Uxy = well function variable
S = storage coefficient
t = elapsed time, in days
x = distance from pumping well of observation well along arbitrarily selected x-axis, in feet
and y = distance from pumping well of observation well along arbitrarily selected y-axis (orthogon 1 to x-axis), in feet

For each of the three wells analyzed, observed drawdown data are matched against type curves to determine values of s, t, W(Uxy) and U(xy). Three equations with three unknowns are then solved simultaneously to determine the transmissivity components Txx, Tyy and Txy. Then principal transmissivities, Tee and Tnn, are calculated from the following equations:

$$Tee = \tfrac{1}{2}\left[(Txx + Tyy) + (Txx - Tyy)^2 + 4Txy^2\right]$$

and

$$Tnn = \tfrac{1}{2}\left[(Txx + Tyy)^2 + 4Txy^2\right]$$

where: Tee = maximum transmissivity
and Tnn = minimum transmissivity

The angle between the arbitrarily selected x-axis and the axis of maximum transmissivity (θ) is then determined by the following equation:

$$\theta = \arctan(Tee - Txx)/Txy$$

D6J.1.5 NEUMAN EQUATION

Theis' equation with Jacob's (1944) correction for aquifer thinning has been used to extensively analyze unconfined aquifer tests. However, this equation does not take into account the free surface boundary of the water table. Theories of unconfined aquifers are more complicated than the Theis equation due to the moving boundary at the phreatic surface. Boulton (1954) presented an unconfined flow equation for drawdown at the free surface. This equation has not been used very extensively, because drawdowns at the phreatic surface and from a well which fully penetrates the aquifer are considerably different. Stallman (1963, 1965) developed type curves for an unconfined aquifer from an electric analog, but these curves have not been used extensively because they are for limited well conditions. Dagan (1967) and Neuman (1972, 1974) have developed computer programs which compute type curve values for unconfined aquifer conditions. Neuman showed that unconfined aquifers have some storage from compression of the aquifer structure and the expansion of the fluid. His equation, therefore, has both a storage coefficient and a specific yield term. Dagan's equation considers only the specific yield for storage. All of these unconfined aquifer equations produce equal type curves for the same conditions except Neuman's curves, which depart from the other curves at early pumping times. Unconfined aquifers which demonstrate the confining effect normally have a flat drawdown curve after the confined portion of the drawdown curve. Finally, the drawdown curve returns to a Theis type drawdown curve. Neuman (1974) and Dagan (1967) have demonstrated that the flat portion of the drawdown curve is due to the vertical flow effects. This flat portion of the drawdown curve will be more obvious as the anisotropic ration (vertical permeability divided by horizontal permeability) decreases.

Development of Neuman (1974) type curves requires execution of a computer program for each individual pump test. Streltsova (1972, 1973) developed an approximation of the vertical flow equation and has shown this approximation is the same as Boulton's (1963) flow equation. Streltsova's approximation allows Boulton's type curves to be used to analyze an unconfined aquifer with consideration of vertical flow, if all wells are fully penetrating. Penetration (the length of the well bore where water enters) of the pumping and observation wells is significant for the pump tests conducted in this investigation. The confining effects of the unconfined aquifer is also important for matching the early drawdown data. Therefore, only Neuman's (1974) method will be further discussed.

Neuman (1974) presents the theory of his unconfined flow equation which is used in the development of Neuman type curves using a computer program. The following is a form of Neuman's unconfined aquifer equation:

$$T = 114.6\,(Q)\,(s_D/s)$$
$$S_y = Tt/\{10{,}770\,(r^2)(t_y)\}$$
$$\beta = (r^2/D^2)(K_v/K_h)$$
$$\alpha = S/S_y$$

where: all terms are the same as previously defined, plus

s_D = dimensionless drawdown (same as well function in Theis equation, except it accounts for penetration and two storage terms)

t_y = dimensionless time (same as 0.25 (1/u) in Theis' equation)

D	=	aquifer thickness, in feet
S_y	=	specific yield
K_v	=	vertical permeability, in feet/day
K_h	=	horizontal permeability, in feet/day

This basic form of the Neuman is used with the geometric setting of the pumping and observation wells and penetration information in the computer program to produce dimensionless drawdown (s_D) versus dimensionless time (t_y) data points for different β (BETA) and α (ALPHA) conditions. Figure J-1 presents the variables used to define well penetrations. The pumping well penetrations are defined by two variables and the observation well's penetration can be defined by two variables which define the top and bottom of the observation well perforation. It can be shown that most observation wells can be represented by a piezometer at the center ZD of the perforated interval without introducing significant errors. The radius of the observation well from the pumping well and the aquifer thickness are included in the BETA term. This term is typically varied for different anisotropic ratios (K_v/K_h). Neuman (1975) recommends the use of a small ALPHA (S/S_y) value for the computer development of the type curves and then adjusting the ALPHA as outlined by Neuman (1975) to obtain the ALPHA value that best fits the observed data.

Neuman's or Dagan's equations do not account for aquifer thinning. Therefore, Jacob's (1944) correction for aquifer thinning is recommended for pump test analyses with these theories also. Pump test data are analyzed by matching the log-log plot of drawdown versus time to Neuman's type curve (s_D vs. t_y) and applying the above equation to the match.

Jacob's straight-line method can be used to analyze drawdown in unconfined aquifers, but the u value is not the only criterion to determine if this method is applicable. A semi-log plot of Neuman's type curves are presented in Figure 2 of Neuman (1975) to demonstrate the applicability of using the straight-line plot to determine transmissivity for unconfined aquifers. Early- and late-time portions of the Theis equation, which form a straight-line, are shown as a solid line on this plot. The straight-line method should yield an accurate transmissivity when the Neuman type curves converge with solid lines. The specific yield value could be in error, however, because partial penetration can cause the late straight line to be shifted parallel to the Theis straight line.

The slope of the straight line from a Neuman type curve is likely to be different from the slope of the Theis straight line. The Theis straight line coefficient of 264 needs to be adjusted to account for the variation in slopes. Therefore, the straight line coefficient adjustment should be made to account for the Neuman unconfined flow theory for the semi-log plots.

D6J.1.6 WTAQ METHOD

The U.S. Geological Survey (USGS) has developed a computer program (WTAQ) to develop type-curves for partially penetrating wells in confined and unconfined aquifers. The unconfined program is based on the Neuman unconfined aquifer equation with a few added features.

The following is the form of the WTAQ equation for fully penetrating wells and an isotropic aquifer using the units of gallons, minutes and feet for the log-log type curve match:

$$T = \frac{114.6Qhd}{s}$$

$$S = \frac{Tt}{10{,}770 t_D r^2}$$

Where parameters are same as above plus:

hd = dimensionless drawdown
t_D = dimensionless time
SIGMA = S/Sy
KV/KH = vertical anisotropic ratio

For semi-log straight-line method:

$$T = \frac{264Q}{\Delta s}$$

$$S = \frac{Tt_o}{1200r^2}$$

Where parameters are the same as above

The unconfined and partial penetration conditions can require that the straight line coefficients (264 and 1200) for the fully penetrating and isotropic need to be adjusted. The slope and intercept of the straight line for the partially penetrating and/or unconfined type curve needs to be compared to the fully penetrating isotropic WTAQ confined type curve to obtain the adjustments in the coefficients.

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