CHAPTER II

Methods of Survey

The methods described in this chapter comprise the specifications for determining the length and direction of lines.

DISTANCE MEASUREMENT

Units

2–1. The law prescribes the chain as the unit of linear measure for the survey of the public lands. All returns of measurements in the rectangular system are made in the true horizontal distance in miles, chains, and links. (Exceptions are special requirements for measurement in feet in townsite surveys, chapter VII, and mineral surveys, chapter X.)

Units of Linear Measure

1 chain = 100 links
= 66 feet
1 mile = 80 chains
= 5,280 feet

Units of Area

1 acre = 10 square chains
= 43,560 square feet
1 square mile = 640 acres

The chain unit, devised in the seventeenth century by Edmund Gunter, an English astronomer, is so designed that 10 square chains are equivalent to one acre. In the English colonial area of the United States the boundaries of land were usually measured in the chain unit, but lengths of lines were frequently expressed in poles. One pole is equal to 25 links, and four poles equal one chain. The field notes of some early rectangular surveys in the southern States show the distance in "perches," equivalent to poles. The term now commonly used for the same distance is the rod.

Land grants by the French crown were made in arpents. The arpent is a unit of area, but the side of a square arpent came to be used for linear description. The Spanish crown and the Mexican Government granted lands which were usually described in linear varas. Both the arpent and the vara have slightly different values in different States. The conversions most often needed are shown in the Standard Field Tables.

Tapes

2–2. Use of the steel tape is the commonly accepted method of measurement. The tapes used vary in length from one to eight chains, the appropriate length depending upon the topography and the nature of the survey. Graduation is to chains and links, and in some instances to tenths of links. For measurements on the slope the vertical angles are determined by use of a clinometer or a transit. The measured slope distances are then reduced to horizontal equivalents by reference to tables or by multiplying the slope distance by the cosine of the vertical angle.

Each tape should be compared with a standard steel tape before being used in the field.

Stadia

2–3. The stadia method is a fast way of making reconnaissance surveys for such purposes as obtaining topography or searching for original corners. Its use is not permitted for measurement of lines. Most transits used by the Bureau of Land Management have a stadia interval with a ratio of 1:132 for use with the chain unit, rather than the standard ratio of 1:100. Data for the reduction of stadia measurements are found in the Standard Field Tables.

Subtense Bar

2–4. The subtense bar may be used provided that no measurement is over ten chains and that the instrument used in connection with it is capable of measuring in single seconds.

Traversing

2–5. Traverses may be run where the terrain is too precipitous for chaining and the intervisible points required for triangulation cannot be practicably obtained. Traversing should be kept to a minimum.

Triangulation

2–6. Triangulation may be used in measuring distances across water or over precipitous slopes. The measured base should be laid out so as to adopt the best possible geometric proportions of the sides and angles of the triangle. If it is necessary to determine the value of an angle with a precision of less than the least reading of the vernier, the method of repetition should be employed.

A complete record of the measurement of the base, the determination of the angles, the location and direction of the sides, and other essential details is entered in the field tablets, together with a small diagram to represent the triangulation.

In the longer and more important triangulations all of the stations should be occupied, if possible, and the angles should be repeated and checked to a satisfactory closure; the latter may be kept within 0′ 20″ by careful use of the one-minute transit.

In line practice the chainmen are frequently sent through for taped measurement over extremely difficult terrain, but with the length of the interval verified by triangulation. This is done to ensure the most exact determination of the length of the line while also noting the intervening topographic data.

Electronic Telemetry

2–7. The measurement of lines by use of electronic telemetry fully meets the requirements for accuracy. Determining factors in its use are the terrain, ground cover, and availability of the proper instruments. Some types are adapted to the measurement of long distances, others to measurement of intermediate distances. Transport and maintenance must be considered in determining whether the use of telemetry will expedite a particular survey. Provision must be made for measuring distances to important items of topography.

The variety of electronic distance-measuring devices, the rapid development of combinations with optical theodolites, and modifications of the instruments make it impracticable to describe the methods of use in this manual. The surveyor should consult the manufacturer's operating manual for calibration, use, care, and adjustments.

A special kind of triangulation is used when it is desired to locate on the ground a point for which the geographic position has been determined in advance. Two intervisible triangulation stations are occupied simultaneously with optical theodolites and electronic measuring devices. A mobile party sets a temporary point at the approximate position of the desired point by reference to a topographic map or aerial photographs. The position of the temporary point is then determined by triangulation or trilateration and the true point is monumented.

The system is made more adaptable by use of the hoversight developed by the United States Geological Survey. This instrument is fixed in a helicopter. The airborne observer is able to identify a point on the ground perpendicularly beneath a flashing target mounted outside the helicopter. The position of the flashing target is then determined by triangulation. Full utilization of the system requires ready contact with computers by telephone or radio. The Airborne Control Survey, as it is called, is carefully planned to coordinate ground crews, helicopter, and computers. Its use has been successful in surveys over extensive areas in Alaska, and experiment is being made in resurveys and in connection with photogrammetric surveys at the present time.

PHOTOGRAMMETRY[1]

2–8. The earliest uses of aerial photography by the cadastral surveyor were for terrain studies, locating himself on the ground, and as an aid in the search for corners. As methodology improved, simple photogrammetric processes enabled the surveyor to delineate topographic features, determine the meanders of water bodies, compute areas of erroneously omitted lands, and lay out townsites as they actually exist. Photogrammetric projects involving both distance measurements and the direction of lines have been completed for both original surveys and resurveys of extensive areas of public lands.

Aerial Camera

2–9. The aerial camera is a high-precision instrument designed for making photographs on which reliable measurements can be made after resolvable errors have been analyzed and removed. The camera must be maintained in calibration at all times. To insure this the calibration should be checked periodically by a competent testing agency such as the Bureau of Standards. The aerial camera used for cadastral surveys should include the following features:

(1) A distortion-free lens with a high resolving power

(2) A between-the-lens shutter

(3) A precision-ground platen

(4) A method of flattening film at the time of exposure

(5) A system of fiducial marks which appear on each photograph and define the lens axis.

Aerial Photography

2–10. An aerial photograph is not a map with a uniform scale throughout, but merely a pictorial representation of the terrain. Geometrically speaking, an aerial photograph is a perspective projection of an area as viewed from a single point above the ground. Relief displacement, lens characteristics, film and paper distortion, and tilt of the camera preclude its having a uniform scale.

Topographic maps may be compiled either by the use of stereoplotting instruments or by making measurements directly upon the photograph. Elements which may affect the accuracy are camera calibration, height of the aircraft above the terrain being mapped, the density and accuracy of ground control, the tip or tilt of the camera at the moment of exposure, film distortion, and the instruments used in making the measurements.

An approximate scale for a vertical aerial photograph is stated by the equation $S = \frac{f}{H\text{-}h}$

where:

f = the focal length of the camera
H = the flying height above sea level
h = the average elevation of the terrain above sea level

Stereophotogrammetry utilizes a stereoscopic plotting instrument (optical-mechanical device) to compile data from aerial photographs. These data, usually in the form of a map, vary in accuracy according to the design of the instrument. Often the instrument embodies a higher degree of accuracy than the photography. By use of a first-order plotting instrument a complete solution of the geometry of the photogrammetric problem may be obtained. All relative displacements of images such as those due to perspective, difference of flying height, lens distortion, and photographic material distortion are resolved.

Several instruments have been designed for aerotriangulation, a method of adjusting consecutive photographs in a strip bridging from one set of control points to another.

Resolution in photography pertains to the sharpness of recording images of two or more light sources, which are close together, so that the images are recognized and distinct. Depending on the characteristics of the lens-film combination, the final resolving power may be enhanced or degraded.

Other factors that affect the quality of the final photograph are aperture opening, distance to the subject, exposure time, atmospheric conditions of haze and brightness of the sun, con-

[1] A complete discussion of the subject in its broad application is contained in the Manual of Photogrammetry, published by the American Society of Photogrammetry.

trast of the subject, vibration, size of the subject, and the processing of the film after exposure.

Field Control

2–11. A network of control points of known position is used as a reference to fix the detail of aerial photographs by photogrammetric processes. The density and distribution of field control points to be photo-identified are determined primarily by the characteristics of the photography, the type of photogrammetric equipment and computer programs to be used, and the accuracy required. Ground control surveys are usually necessary to identify the existing basic control and to provide additional control points.

The basic control into which the supplemental surveys are tied, the supplemental surveys themselves, and the photo-identification of points must in toto provide the degree of accuracy required of the resultant cadastral survey. The survey methods used in the control survey have to be of equal or higher order accuracy than is specified for the results. The classification and standards of accuracy of geodetic control surveys are outlined in Bureau of Budget (now Office of Management and Budget) Circular A–16. The density, spacing, accuracy, and manner of marking of photo-identified points must conform to the criteria established by the responsible photogrammetrist.

Datum. A basic network of high-order geodetic control has been established throughout the United States. This network has been developed by combining a number of separate geodetic datums into a single datum known as the North American Datum of 1927. All horizontal contol stations established for photogrammetric projects should use this datum as the base for computing values for the control stations. (section 2–82.)

Vertical control in the United States is referred to an arbitrary level for the entire nation which was based on mean sea level as determined by observations made over a period of years at tidal stations on the Atlantic, Pacific, and Gulf Coasts. Several adjustments have been made of the basic network, the most recent in 1929.

Basic vertical control bench marks within or adjacent to a photogrammetric project should be used to expand the vertical control over the project area. When there is no basic vertical control near the project area, an arbitrary datum may be assumed and expanded to control the project.

State Plane Coordinates. State plane coordinate systems are used extensively for photogrammetric plotting. (section 2–83.) Formulas and tables for computing values for these systems have been prepared by the United States Coast and Geodetic Survey (now the National Geodetic Survey) for each individual State. The computations involve corrections for grid lengths, sea level factors, and grid azimuths.

Horizontal Control. Basic horizontal control is that which has been established by the National Geodetic Survey to form the National Network; this should be the origin for all supplemental control on each photogrammetric project.

The supplemental control should be of sufficient density to permit an efficient control of all the photographs at the time of the analytical phototriangulation. The density of control will vary with the size of the project, but generally horizontal control should be located in every six to eight models in a bridge with never fewer than four control points in a flight strip.

The supplemental control may be established by triangulation, trilateration, or traverse using transit and tape or theodolite and electronic distance-measuring instruments. In all applications the control thus established should be executed using second-order methods and meet second-order accuracy.

Vertical Control. The basic vertical control network is that established by the National Geodetic Survey by spirit leveling; when such control is within the project area, it should be used to establish supplemental vertical control.

The supplemental vertical control may be established on an assumed datum where no basic control network is near the project.

The supplemental control may be established by spirit leveling or trigonometric leveling.

Both the horizontal and vertical control points are normally targeted prior to the aerial photography. The targets are centered over the respective stations and have a symmetrical design easily identified on the photography. Care should be taken when selecting the size, shape, color, and material to be used for the targets.

In cases where targeted points have been destroyed prior to photography it may be necessary to substitute "natural targets" to supplement the control. Such points are selected in the field and referenced into the control scheme. The identification should be made only while viewing the photography stereoscopically and at the site of the feature. A photograph showing the feature and its relationship with the destroyed station should be furnished the operator of the comparator at the time of the analytical phototriangulation.

Mechanical Phototriangulation

2–12. The mechanical (known also as analogue or instrumental) method of phototriangulation establishes positions and elevations by use of an instrument viewing a spatial model. Precise connections are made between successive models which in turn are tied to vertical and horizontal control. After adjustment, an accurately scaled representation of the project area can be depicted. This method has been used for several successful cadastral survey projects.

Analytical Phototriangulation

2–13. Analytical phototriangulation is a mathematical determination of ground positions of panelled points observed in a strip or block of aerial photographs. The positions are determined by use of electronic computers and are based on coordinate measurements of the image positions in each photograph. The method considers such factors as camera calibration, film distortion, atmospheric refraction, and earth curvature during the computations.

The instruments used to determine photographic coordinates, from which the ground positions are established, are the comparator (either monocular or stereoscopic), point-marking and transfer devices, and computers. The advent of the electronic computer made it practicable to use analytical methods in phototriangulation. The basic foundation for analytical photogrammetry had been established by Sebastian Finsterwalder about 1900.

The accuracy of the data obtained by use of the analytical process is usually of a higher order than that obtained by the mechanical methods. The Bureau of Land Management has therefore adopted it for use in photogrammetric cadastral surveys.

Photogrammetry in Original Surveys and Resurveys

2–14. Pilot projects employing photogrammetric methods for making original surveys and resurveys have led to standardization of methods and equipment. As new equipment and refinements in methods are developed they will be tested and employed as warranted.

Protraction Diagrams. A diagram representing the plan for the extension of the rectangular system over unsurveyed public lands, based upon computed values for the corner positions, is termed a protraction diagram. Such diagrams have been prepared for substantially all unsurveyed areas of public lands except the Aleutian Islands and southeastern Alaska.

A successful photogrammetric project requires a good plan and coordination between the cadastral surveyor and the photogrammetrist. As in any cadastral survey, all pertinent data must be reviewed, including maps, aerial photographs, geodetic surveys, cadastral surveys, and protraction diagrams.

Certain basic steps are always required to complete a photogrammetric survey:

Original Surveys

Corner Positions. The theoretical corner points are first plotted upon a map or on existing aerial photographs at the coordinate positions of the protraction diagram or other plan. The transfer may be accomplished by scaling on the map, or the positions may be located by using plotting instruments such as the Kelsh plotter. The more accurately these values can be plotted, the smaller the moves will be from the panel points when permanent corner monuments are established.

Control. The plan of geodetic control depends on the number of flight lines and the number of models in each flight line. It should be based on triangulation or traverse stations established to second-order accuracy. It is advisable that electronic distance-measuring instruments and theodolites be used in establishing new stations. Final values for such stations should be given as state plane coordinates.

Panels. The theoretical position of each corner, as plotted on maps or existing photographs, as well as each original or new control station, is marked on the ground by a systematically designed panel. Care should be taken to center the panel over the monument or survey stake.

The panels should be of such a design and size as to be conspicuous in the subsequent aerial photographs. The photography is undertaken immediately following the control and paneling operation in order to assure the least disturbance to panelled points. If a panel is destroyed before the photography can be accomplished, the photogrammetrist should select a natural object near the destroyed panel to serve as a substitute during the remaining operations.

Aerial Photography. Complete stereoscopic coverage of the area to be surveyed is essential. The photography should have a minimum of 55% forward lap and 30% side lap between flight lines. With a photoscale of 1:20,000, it is possible to perform surveys having a root-mean-square horizontal error of plus or minus one foot.

In certain areas it is advisable to use several types of aerial negatives—color, panchromatic, and infrared false color—for positive identification of panelled points. Experiments are being conducted to determine the best film-filter-background combination to give maximum clarity in delineating panelled points.

Analytical Bridge. An analytical bridge, a form of phototriangulation, establishes coordinate values for the panelled points and also natural objects. The photographic coordinates are transformed into State plane coordinates, which are used in computing corner moves.

In addition to panelled points, it is necessary to obtain positions of houses, windmills, or other features that require ties to complete the cadastral survey. The cadastral surveyor must work closely with the photogrammetrist to assure that the necessary ties to such items are made.

Once the corners are monumented at the protracted positions, the cadastral surveyor prepares his plats in the normal manner, prefacing his notes with a statement concerning the method of procedure.

Execution of Resurveys

As in the making of original surveys, planning and cooperation between the cadastral surveyor and the photogrammetrist are essential to success.

Corner Positions. From the original survey notes and plats the theoretical position of each previously established corner is plotted upon existing aerial photography. Where suitable maps do not exist, a cursory search for the exterior boundaries of the townships to be resurveyed should be made. Any corners found are identified upon existing aerial photography and the interior corners plotted in accordance with the record of the original survey.

A careful search is made for the corners in the positions plotted on the photographs. When a corner is recovered, it is rehabilitated or remonumented. In the event that no positive evidence of the corner is recovered, a temporary stake is set at the theoretical position. In either case a State plane-coordinate value is established to be used in control for further search or in computing subsequent corner moves.

Geodetic control, aerial photography, and analytical bridge methods and procedures are the same as for original surveys.

Meanders

The sinuosities of a shoreline may be produced on a manuscript base by use of a stereoplotting instrument. The plotting scale is usually five times that of the photography. Angle points are selected along the shore, and the coordinate values of both meander corners and angle points are determined by scaling. Courses and distances of the meanders are computed from the coordinate values. The accuracy of the results depends on the accuracy with which the meander corners are photo-identified, the precision with which the sinuosities of the shoreline are drawn, and the correctness of scaling from the manuscript.

In areas of little relief single photographs can be used as the displacement of features is at a minimum. Distortion caused by camera tilt is small enough to be removed by adjustment. Either contact prints projected by a reflecting projector and enlarged to convenient scale or enlargements made from the original film may be used.

Whether the stereoplotting instrument or the single print is used, it is desirable that the field man verify the shoreline and perhaps delineate it on the photographs with colored ink.

Accuracy Checks

2–15. In both the original survey and the resurvey it is advisable to establish the coordinate position for a number of corners selected at random, being sure that there are several check positions in each flight line. It is preferable that these check positions fall in the overlap area between flight lines, this being one of the weak points in a photogrammetric bridge.

The values of the check points should be withheld from the original bridge and used as a check. If they fall within the allowable tolerance for accuracy, they will have served their purpose. If they do not meet the accuracy tolerance, the bridge may be strengthened by using them as control. In such a case additional accuracy checks will be required to assure that the survey meets the necessary standard.

Where photogrammetry is used as a means of establishing the position of section corners, accuracy is necessarily stated as a radius of error rather than a ratio of closure, since any position established is independent of monuments preceding or following along a boundary. Errors of position are not accumulative, and a stated radius of error means that any monument's position may differ from the protracted value by the full radius of error and in any direction from the protracted point. Since the acceptable radius of error is the same for each bridged point, the error in bearing and the percentage error in distance between two survey monuments will vary inversely as the length of the course.

Accuracy

2–16. The accuracy obtainable in photogrammetric surveys depends on the scale and type of photography, the instruments used, the skill of the compiler, the density of ground control, the amount of relief, and the nature of the vegetative cover. These factors relate to the data taken from the photographs. If markers are positioned by relationship to nearby photo-identifiable objects, the precision of the field methods used also affects the final accuracy. If meanders are recorded, the reliability of their delineation on the photography is a factor in the accuracy of the work.

It is axiomatic that "the greater the accuracy, the greater the cost." The scale of the photography for each project, therefore, should be commensurate with the accuracy required. The amount of topographic relief may affect the choice of methods. In flat terrain, with photography nearly vertical, measurements for some purposes may be made on a photographic print. As the relief or the tilt increases, rectification and adjustment are necessary.

If precision is not required, a tube magnifier, which can be carried in the pocket, will measure 0.005 inch. A precision comparator has a least measurement of one micromillimeter. At a photo scale of 1:20,000, 0.005 inch represents eight feet, while one micromillimeter represents 0.06 foot. These figures are cited to illustrate how methods and instruments can be selected to give desired precision in results.

THE DIRECTION OF LINES

2–17. The direction of each line of the public land surveys is determined with reference to the true meridian as defined by the axis of the earth's rotation. Bearings are stated in terms of angular measure referred to the true north or south.

2–18. *The Magnetic Needle.* The Manual of 1890 prohibited the use of the magnetic needle except in subdividing and meandering, and then only in localities free from local attraction. The

Manual of 1894 required that all classes of lines be surveyed with reference to the true meridian independent of the magnetic needle.

A field note record is required of the average magnetic declination over the area of each survey. The value is shown on the plat and in the field notes. The principal purpose of this record is to provide an approximate value for use in local surveys and retracements, where a start is to be made by the angular value of the magnetic north in relation to the true north.

Methods of Establishing Direction

2–19. Current practice is to determine true azimuth by one of the following methods:

(1) Direct observations of the sun, Polaris, or other stars

(2) Observations with a solar attachment

(3) The turning of angles from triangulation stations of the horizontal control network.

At remote locations, if these methods are made impracticable for long periods by thick cloud cover, angles may be turned from identifiable lines of an adjacent Bureau of Land Management survey. Use may also be made of a gyro-theodolite, properly calibrated and previously checked on an established meridian.

Observations—General Considerations

2–20. *Sequence of Observations.* A small error in latitude or azimuth has only a slight effect in time. When all three are unknown, the order of sequence in their determination should be (1) time, (2) latitude, and (3) azimuth.

2–21. *Geographic Position.* The longitudes that are shown upon maps refer to the zero meridian of the Royal Observatory at Greenwich, England. The map values for longitude scaled from the topographic maps of the United States Geological Survey may be accepted for use in making any of the calculations incident to the observations for time, latitude, and azimuth that are required with Manual practice. Where these maps are not available, it is probable the surveyor will be able to find others that will show longitude within the degree of accuracy required. Precision in both latitude and longitude may be secured wherever geodetic stations have been established.

The showing of latitude and longitude on the plat of the cadastral survey should be extended to seconds if ties to a geodetic station warrant that refinement.

2–22. *Precision of Observations.* The methods that are set out in the Manual for a well balanced observing program are good for results within ± 6 seconds of time and $\pm 15''$ in latitude and azimuth, when estimated vernier readings are made to the nearest $30''$.

2–23. *Astronomy in the Manual.* The basic astronomy needed for understanding of the observations described in the Manual is well covered in college courses in applied astronomy. The theory relating to the observations and the derivation of formulas is not repeated in the Manual. The subjects are treated with a view to securing the most direct practical results. The methods are not difficult when coupled with practice in making the observations. Until the steps become familiar it is helpful to record for an experienced observer and to assist in making the reductions.

The methods applied principally in observations upon Polaris and the sun are arranged to facilitate the work under most conditions encountered in the field. The tables and formulas that are published in the Standard Field Tables and in the Ephemeris are designed for the convenience of the cadastral surveyor in the field.

The bright stars in the equatorial belt may be observed to secure refinements and to verify results secured by observations on the sun and Polaris. These stars may be selected for favorable position in declination at any date when the sun is either too low or too high for the desired observation. The south declination stars are needed for certain observations in Florida, the higher north declination stars in Alaska. The stellar methods are indispensable to a well balanced observing program whenever high precision is required.

2–24. *Symbols.*

$\neq$: The symbol for inequality, which is here used to show a relation that approaches equality.

v: Observed vertical angle. In altitude observations on the sun the reductions to the sun's center both vertically and horizontally, as well as instrument errors, are compensated by tak-

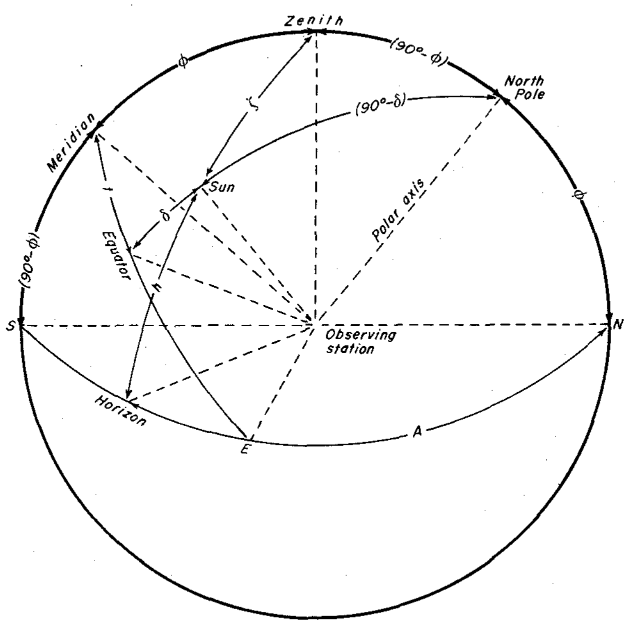


FIGURE 3.—The "pole-zenith-sun" triangle as viewed from outside the celestial sphere.

ing direct and reversed observations on the opposite limbs of the sun. The mean observed vertical angle to the sun's center is designated v in the notation. In single observations the vertical reduction to the sun's center = 16′. A refinement is had by referring to the Ephemeris for the value of the sun's semi-diameter for the date of observation.

h: True vertical angle to the sun's center, or to Polaris or any other star. In altitude observations, h = v — refraction in zenith distance. A refinement is had in altitude observations on the sun by adding the value of the sun's parallax = 8″.9 cos v, opposite in effect to refraction, which results from the observer's position above the center of the earth.

ζ: Zeta: True zenith distance of the sun's center, and to any star: equal to 90° — h.

ϕ: Phi: Latitude of the station of observation.

λ: Lambda: Longitude of the station of observation.

δ: Delta: Declination of the sun, Polaris, or any star, to be taken from the Ephemeris for the date of observation.

A: Azimuth angle from the true meridian to the sun's center, or to Polaris or any other star when making an azimuth observation.

2–25. *Refraction.* Tables of mean refractions both in zenith and polar distance appear in the Standard Field Tables, arranged to meet the requirements of field use. Another table lists coefficients to apply to mean refractions in zenith or polar distance for variations in atmospheric pressure and temperature. Lacking a barometer to determine atmospheric pressure, the argument "approximate elevation above sea level" may be substituted. The differences between the true and the tabulated refractions are generally small and negligible excepting for the combined effect of low vertical angle with high elevation or extreme temperature. The following example shows the method to be used in reductions from the tabulated refractions:

Tabulated refraction = 6′45″ or 6′.75

Elevation above sea level = 10,000 feet, coefficient for which is 0.70

Temperature at time of observation = 82° F., coefficient for which is 0.94

True refraction = 0.70 × 0.94 × 6′.75 = 4′.44 or 4′26″

Time

2–26. Because the earth revolves around the sun, a point on the earth's surface faces the sun one less time each year than it does the other stars. There are, therefore, two normal time rates, solar time and sidereal time. Solar time is divided into three distinct classes—apparent time, local mean time, and standard time.

2–27. *Apparent time* is based upon the *real sun,* the 24-hour period of which counts from the sun's meridian passage of one day—apparent noon—to the next meridian passage. This rate is irregular. A sun dial shows apparent time. A watch may be set to read nearly correct apparent time for the day or approximate apparent time for several days, but it will need changing from week to week because of the irregular rate.

2–28. *Mean solar time* is based upon a fictitious or imaginary sun, whose solar day is a mathematically uniform 24 hours. The ordinary watch measures mean time.

2–29. *Local mean time* is identical with mean solar time on the meridian at the station where that time is being employed. It is correct on that meridian only. Stations that are 1° apart in longitude differ by four minutes in local mean time, one hour for 15° difference in longitude.

2–30. *The equation of time* is the amount to be added to, or subtracted from apparent time to convert over into local mean time. The equation of time is changing constantly. Its value for apparent noon each day, on the Greenwich meridian, is tabulated in the Ephemeris. The equation of time reaches a maximum of about 16 minutes early in November.

2–31. *Standard time* is identical with local mean time on the central meridian of each time belt, as Eastern Standard Time on the 75th meridian; Central Standard Time on the 90th meridian; Mountain Standard Time on the 105th meridian; Pacific Standard Time on the 120th meridian; Yukon Standard Time on the 135th meridian; Alaska Standard Time on the 150th meridian; Bering Standard Time on the 165th meridian of longitude. Correction for longitude is all that is required for converting over into local mean time, additive when east of the central meridian, subtractive when west. An additional correction of one hour is necessary when "daylight saving" time is in effect.

2–32. If an observation is to be made of Polaris on a different meridian than that of the preliminary observation for time, it is important to adjust the local mean time to the new station. This adjustment amounts to 23 seconds across one township at Cape Sable, Florida; 60 seconds at Point Barrow, Alaska. It is 30 seconds across one township in latitude 46°. For example, a Polaris observation is to be made at a station in latitude 46°; the adjustment in local mean time, for longitude, for the time observation that may not be made in that same meridian will be at the rate of 5 seconds per mile. A watch that reads correct local mean time at the point of time observation will be $0^m\ 5^s$ *slow* of local mean time in the meridian one mile to the east, or the same amount *fast* for the meridian one mile to the west. This adjustment may be allowed for when the time observation is made somewhere on the line of the survey and the Polaris observation is made at field party headquarters.

The unit of sidereal time is measured by one revolution of the earth on its axis, the 24-hour period of which is equivalent to 23 hours 56 minutes 4.091 seconds in mean solar time. There are 366¼ sidereal 24-hour periods in the solar year of 365¼ days.

The mathematical equations that are employed in the observations upon the equatorial stars, for time and altitudes, and for the azimuths and altitudes of Polaris at various hour angles, are based upon the sidereal time rate. The same equations are applicable in the reduction of observations upon the sun for time, the moment of the observation being expressed in apparent time.

Assume that a star and *mean sun* cross the Greenwich meridian at the same instant. The star would cross each succeeding meridian ahead of the sun by an increasing time interval proportionate to the longitude west of Greenwich. These time intervals, called *sidereal conversions,* are listed for increasing longitudes in both the Ephemeris and the Standard Field Tables. Sidereal conversions are applied to the mean solar time to obtain sidereal time and vice versa.

Sidereal time is not employed directly in the Manual methods. It is avoided through the plan of the tabulations that are published in the Ephemeris for the upper culmination and elongation of Polaris, and for the transit (meridian passage) of the equatorial stars, which are given in terms of mean solar time, Greenwich meridian, for the ordinary civil date, a.m. or p.m. The azimuths and altitudes of Polaris are tabulated in terms of mean time hour angle.

2–33. In the entry of the record of an observation, the watch time is the reading at that moment. The watch may be set to read the approximate local mean time, or it may be set to carry the approximate standard time. In either case the "watch error" is the difference between the actual reading and what would be the exact local mean time or standard time as intended. The watch error in standard time may be determined by comparison with a clock that reads the correct standard time controlled electrically, or the comparison may be made with the radio time signals.

There is usually a personal preference as to the setting of a watch. Many prefer to set to standard time. Others on extensive field work find it convenient to change over to local mean time, or to carry a substitute watch set to local mean time. On solar transit orientation, the time circle reads apparent time. If the solar unit is being used constantly, as is nearly always the case where the line runs through heavy forest cover or dense undergrowth, many surveyors like to use a watch set to apparent time.

The record entry should therefore be explicit (1) as to the setting of the watch to approximate standard, local mean, or apparent time; (2) the conversion, if from standard to local mean time; and (3) the method of ascertaining the watch error in terms of local mean time in every case when making an hour angle observation on Polaris. Many Polaris observations are made during the season, sometimes daily. It is for this purpose that the Manual devotes so much attention to the practical field observations for time.

2–34. The element of time enters into all azimuth determinations, apparent time for all observations upon the sun, local mean time for all observations on Polaris and other stars. The sun's declination varies with the apparent time and the longitude west from Greenwich. The declination enters into all observations on the sun for azimuth. Thus the apparent time and longitude should be known to a degree of accuracy commensurate with the refinement necessary in computing the sun's declination. The azimuth of Polaris varies with local mean time of observation, which must be known to a degree of accuracy consistent with the result wanted in the determination of the true meridian.

2–35. In observations on Polaris at elongation, precision in local mean time is unnecessary, but in hour angle observations upon Polaris it will be noted that at upper or lower culmination, in latitude 40° for example, Polaris varies 1′ in azimuth in about three minutes of time. This interval of time slowly increases toward elongation, and in the latter position more than 30 minutes of time are required for a change of 1′ in azimuth.

Examples of Time Conversions

2–36. *Standard time into local mean time:* Watch reading ± watch error in standard time ± correction for longitude. The correction for longitude is additive east and subtractive west of the standard meridian of the time belt. The conversion table, "degrees to time," Standard Field Tables, is convenient in this reduction. For example, in longitude 77°01′37.5″ W.:

Watch time of observation	= $6^h26^m40^s$ p. m.
Watch slow of 75th meridian standard time by comparison with a standard clock	= $+1^m22^s$
Correction for longitude of station (77°01′37.5″ W. or $5^h08^m06.5^s$)	= -8^m06^s
Local mean time of observation	= $6^h19^m56^s$ p. m.

2–37. *Apparent time into local mean time:* Apparent time of observation ± the equation of time. The equation of time is taken from the Ephemeris for the date of observation and corrected for the longitude and time of observation, conveniently interpolated as the interval from Greenwich noon to the time of observation. The watch error in local mean time is then found by taking the difference between the watch reading at the instant of the observation and the reduced mean time of observation. For example, in longitude 77°01′37.5″ W.:

Mar. 18, 1970, apparent time of altitude observation upon sun		= $3^h42^m11^s$ p. m.
Equation of time, Greenwich apparent noon	$+8^m12.5^s$	
Interpolation for longitude of station 5^h08^m W., and time of observation 3^h42^m, p.m., 8^h50^m after Greenwich noon, or 8.83/24 of change (17.64^s) in 24 hours =	-6.5^s	
Equation of time	$+8^m06.0^s$	$+8^m06^s$
Local mean time of observation		= $3^h50^m17^s$
Watch time of observation		= $3^h57^m53^s$
Watch fast of local mean time		= 7^m36^s

This reduction is made when the apparent time has been determined by solar observation as in section 2-63. If the correct watch time is known, a reverse process is used to convert local mean time to apparent time when computing the declination of the sun at the time of an observation.

Polaris

2–38. Polaris, the North Star, occupies a position in the northern heavens about 1° from a line defined by the axis of the earth's rotation. Being a star of the second magnitude and near the polar axis, it ranks as the most useful circumpolar star. It will be assumed that the surveyor has learned how to identify Polaris in the clear night sky by reference to the "pointers" in the constellation of the "Great Bear," popularly called the "Dipper." Polaris, α Ursae Minoris, is nearly on a line (or great circle) determined by the pole and the star δ Cassiopeiae. Both stars are located in the same direction from the pole. The same line, or great circle, passes near the star ζ Ursae Majoris, another star of the "Dipper." The latter star is located on the *opposite* side of the pole. The relative position of the three stars gives an immediate indication of the approximate position of Polaris in its diurnal circle at that time. The three stars are all of about the same brightness. Instructions will follow regarding the identification of Polaris by instrumental methods during the twilight period, before the star is visible to the naked eye. The same method may be used for verification of a night observation if the neighboring constellations are obscured by clouds.

An experienced surveyor can readily observe Polaris at sunrise or sunset, reading the measurements without artificial illumination, and with a very clear atmosphere can make the observation when the sun is as much as 20 or 30 minutes above the horizon.

Polaris has a diurnal circle about the earth's polar axis similar to the diurnal circle of other stars, though Polaris has the smallest circle of any naked-eye star. The daily circuit of Polaris is covered in one sidereal day of 24 sidereal hours, or an equivalent of 23 hours 56 minutes 4.09 seconds of mean solar time. In its diurnal circle Polaris crosses the meridian twice, once at upper culmination, or above the polar axis, and once at lower culmination, or below the polar axis. The direction of the apparent motion of Polaris is suggested by the following diagram:

The pointings of the arrows on the circle (at right) indicate the direction of the apparent motion of Polaris in its diurnal path. The point-

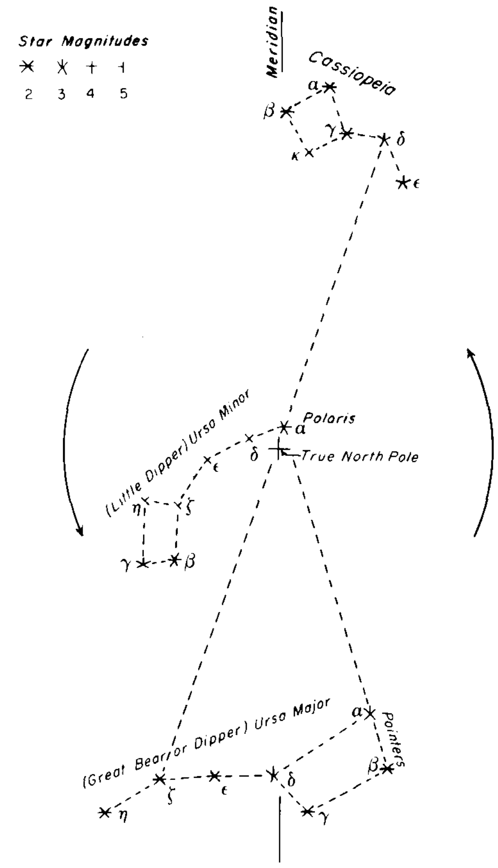


FIGURE 4.—Naked-eye identification of Polaris.
About noon March 23d.
About 6 a.m. June 22d.
About midnight September 22d.
About 6 p.m. December 22d.

ings of the arrows on the lines tangent to the circle show the direction of travel at the epochs of culmination and elongation. If the surveyor has any doubt in regard to the quadrant occupied by Polaris in its diurnal circle at the

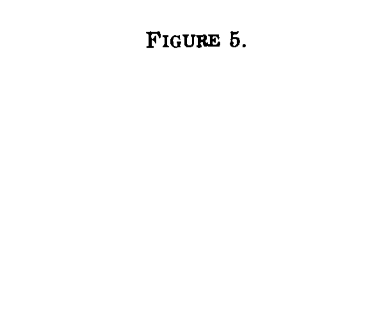


FIGURE 5.

time of an observation, he may set the intersection of the telescope cross-wires exactly on the star, then, without moving the instrument, note the direction of the star's motion and compare with the diagram.

The motion of Polaris at western elongation is vertically downward; at eastern elongation the motion is vertically upward. At western or eastern elongation the motion in azimuth is zero.

At the equator, if Polaris could be observed, the hour angle of Polaris at elongation would be 90°0′0″ or $6^h0^m0^s$ sidereal hour angle or 5^h59^m-1.02^s mean time hour angle, but as stations of observation are occupied in the higher latitudes the hour angle of Polaris at elongation decreases progressively. The reason for this is that all vertical planes intersect at the zenith, and the point of tangency of a vertical plane with the diurnal circle of Polaris occurs at points corresponding to decreasing hour angles with the higher latitudes. The spread of the two vertical planes intersecting Polaris at eastern and western elongation increases with higher latitudes, giving increasing azimuths at elongation with the more northern latitudes.

2–39. The position of Polaris in its diurnal circle at any time may be determined by reference to the mean time interval from upper culmination to any observed position west of the meridian, or by reference to the mean time interval from any observed position east of the meridian to the succeeding upper culmination.

The Greenwich mean time of upper culmination of Polaris is tabulated in the Ephemeris for every day in the year, arranged for the ordinary civil date, a.m. or p.m.

Local mean time of upper culmination of Polaris: the Greenwich mean time of upper culmination of Polaris is to be taken from the Ephemeris for the date of observation. The amount to be subtracted from the Greenwich mean time of upper culmination of Polaris to obtain the local mean time of upper culmination, in which the argument is the longitude west from Greenwich, is obtained from the table of sidereal conversions in the Standard Field Tables without computation.

Example of reduction from the Greenwich mean time of upper culmination of Polaris to

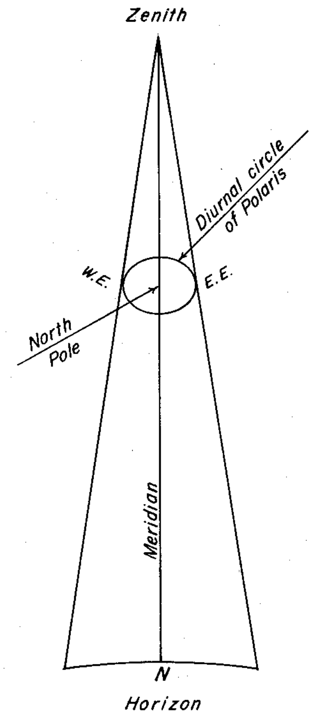


FIGURE 6.—The meridian and vertical planes tangent to the diurnal circle of Polaris as viewed from inside the celestial sphere.

the local mean time of upper culmination of Polaris, longitude 111°15′ W.:

Aug. 16, 1972, Gr. U. C. of Polaris	= $4^h27.3^m$ a.m.
Red. to long. 111°15′ W., 1^m13^s	= −1.2
L. M. T. of U. C. of Polaris	= $4^h26.1^m$ a.m.

The local mean time of the meridian passage of any other star is reduced in the same way from the time for the Greenwich meridian to the longitude of the station. It should be noted that this conversion is at the rate of approximately 10 seconds of time for each 15° (or one hour) of longitude, subtractive to the west. Also, the meridian passage of each star comes approximately four minutes earlier each succeeding day in terms of local mean time. On one calendar day each year a star will have a double meridian passage.

2–40. The Greenwich mean time of elongation of Polaris, latitude 40°, is tabulated in the Ephemeris for every day in the year, arranged for the ordinary civil date, a.m. or p.m. This is reduced to the local mean time for the position of the station, in two steps: first, for longitude; second, for latitude.

Local mean time of elongation of Polaris: the mean time of elongation of Polaris, Greenwich meridian, latitude 40°, is taken from the Ephemeris for the date of observation. The amount subtracted from the mean time of elongation of Polaris, Greenwich meridian, latitude 40°, to obtain the mean time of elongation of Polaris, local meridian, latitude 40°, in which the argument is the longitude west from Greenwich, is obtained from the table of sidereal conversions, Standard Field Tables, without computation. The amount to apply to the local mean time of elongation of Polaris latitude 40° to obtain the local mean time of elongation of Polaris at the latitude of observation is tabulated in the Ephemeris in connection with the table of azimuths of Polaris at elongation.

Examples of reduction from the Greenwich mean time of elongation of Polaris, latitude 40°, to the local mean time of elongation of Polaris, latitude 64°30′ N., and longitude 146° 30′ W.:

Eastern Elongation

Sept. 14, 1972, Gr. E. E. of Polaris, lat. 40°	= $8^h33.8^m$ p.m.
Red. to long. 146°30′ W., 1^m36^s	= −1.6
Red. to lat. 64°30′ N.	= +4.3
L. M. T. of E. E. of Polaris	= $8^h36.5^m$ p.m.

Western Elongation, same station

Oct. 31, 1972, Gr. W. E. of Polaris, lat. 40°	= $5^h25.7^m$ a.m.
Red. to long. 146°30′ W., 1^m36^s	= −1.6
Red. to lat. 64°30′ N.	= −4.3
L. M. T. of W. E. of Polaris	= $5^h19.8^m$ a.m.

2–41. *Hour Angles.* The interval between the time of a star's meridian passage (or transit) and other position in its diurnal circle is termed the star's hour angle. This is measured in sidereal time, or the equivalent in angular measure in degrees, minutes, and seconds. The hour angle may count either to the east or to the west of the meridian. As the ordinary watch is rated in mean solar time, the observation for time, and the reductions, require the conversions from the one rate to the other. The Ephemeris and the Standard Field Tables both include a table of sidereal conversions. The conversion increment is to be subtracted from a sidereal interval, or added to a mean time interval, to obtain the equivalent. The conversion is required in the reduction of an altitude observation upon a star for time, as the observed hour angle is in the sidereal interval.

Conversion of a mean time interval into a sidereal time interval, or vice versa: The amount to apply to one time interval to obtain the other time interval is found in the table of sidereal conversions without computation.

Example of conversion of a mean time interval into a sidereal time interval:

Mean time hour angle of Polaris for an assumed observation in Alaska = $7^h32.6^m$	= $7^h32^m36^s$
Conversion into equivalent sidereal hour angle	= +1 14
Sidereal hour angle	= $7^h33^m50^s$
7^h	= 105°
33^m	= 8°15′
50^s	= 12′30″
Sidereal hour angle converted to degrees	113°27′30″

The conversion from a mean time interval to the equivalent sidereal hour angle is required in the analytical reduction of the hour angle observation upon Polaris for azimuth or latitude, whenever the reduction is made by the equations in place of, or as a check upon, taking the values from the tables of azimuths and altitudes that are published in the Ephemeris. The conversion is not required if the tables are employed, as the values are tabulated in mean time hour angle.

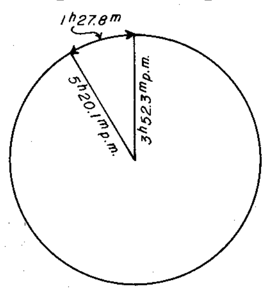
2–42. *Hour angles of Polaris:* a mean time hour angle of Polaris *west* of the meridian is the mean time interval *from* the local mean time of the last preceding upper culmination *to* the local mean time of observation of Polaris. A mean time hour angle of Polaris *east* of the meridian is the mean time interval *from* the local mean time of observation *to* the local mean time of the next succeeding upper culmination of Polaris.

The above application of the term "hour angle" is a departure from conventional usage, which is employed in order to simplify the steps. One step relating to hour angles for positions east of the meridian is avoided. Polaris crosses the meridian at lower culmination at an hour angle of $11^h58^m02^s$. In the arrangement of the various examples, the observations west of the meridian have been referred to the last preceding upper culmination, those east of the meridian have been referred to the next succeeding upper culmination, thus avoiding any hour angles exceeding $11^h58^m02^s$.

Hour angles of Polaris: verification by the star chart: a simple check on the approximate value of the hour angle at any moment, any date, and of the position west or east of the meridian, may be secured by use of the star chart in the Ephemeris. First, scale a line for the date, then place the overlay scale on the chart making the date line agree with the scale for the time of observation, a.m. or p.m., lower set of figures. In this position, note where Polaris will be found with respect to the meridian line of the overlay scale. Finally, read the scale for hour angle, upper set of figures, star west or star east of the meridian. The reduction values should of course be taken from the tabulated daily position of Polaris.

The tables of the azimuths of Polaris at all hour angles, that are published in the Ephemeris, are tabulated with the argument in *mean time hour angle,* counting from upper culmination. Therefore, for an observation west of the meridian the hour angle is referred to the preceding upper culmination; for one east of the meridian the reference is to the next succeeding upper cumination. The hour angle at lower culmination is the half (11^h58^m) of the sidereal day ($23^h56.1^m$). That position is a good one for a latitude observation. It should be understood that hour angle observations for azimuth are not referred to the point of lower culmination. The equations for the azimuth and altitude observations count strictly from upper culmination.

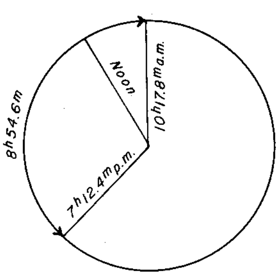
Examples of computing hour angles of Polaris, all for long. 117°15′ W.:



(1)

West of the meridian, p.m. observation, U.C. in p.m.

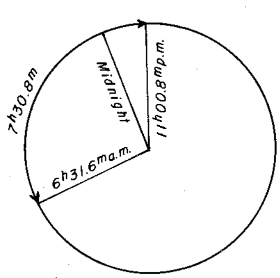
Feb. 23, 1972, l.m.t. of observation		$= 5^h20.1^m$ p.m.
Gr. U.C., same date	$= 3^h53.6^m$ p.m.	
Red. for long. (sidereal conversion)	$= -1.3$	= 3 52.3 p.m.
Hour angle, west		$= 1^h27.8^m$



(2)

West of the meridian, p.m. observation, U.C. in a.m.

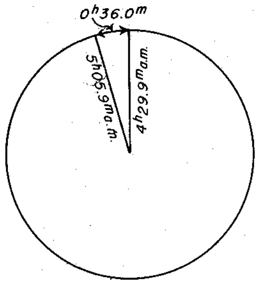
May 18, 1972, l.m.t. of observation		= {+12 $7^h12.4^m$ p.m.
Gr. U.C., same date	$= 10^h19.1^m$ a.m.	
Red. for long.	$= -1.3$	= 10 17.8 a.m.
Hour angle, west		$= 8^h54.6^m$



(3)

West of the meridian, a.m. observation, U.C. in p.m.

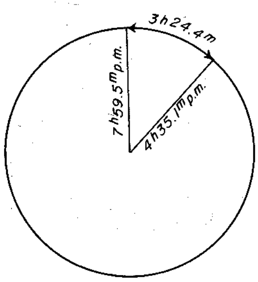
Nov. 7, 1972, l.m.t. of observation		= {+12 $6^h31.6^m$ a.m.
Gr. U.C., Nov. 6	$= 11^h02.1$ p.m.	
Red. for long.	$= -1.3$	= 11 00.8 p.m.
Hour angle, west		$7^h30.8^m$



(4)

West of the meridian, a.m. observation, U.C. in a.m.

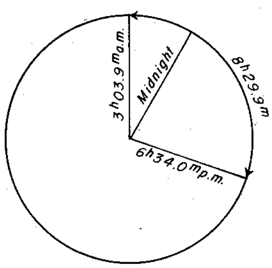
Aug. 15, 1972, l.m.t. of observation		= $5^h05.9^m$ a.m.
Gr. U.C., same date	= $4^h31.2^m$ a.m.	
Red. for long.	= −1.3	= 4 29.9 a.m.
Hour angle, west		= $0^h36.0^m$



(5)

East of the meridian, p.m. observation, U.C. in p.m.

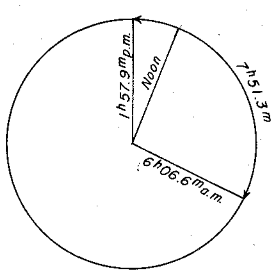
Gr. U.C., Dec. 22, 1972	= $8^h00.8^m$ p.m.
Red. for long.	= −1.3
L.m.t. of U.C., Dec. 22	= 7 59.5 p.m.
L.m.t. of observation, same date	= 4 35.1 p.m.
Hour angle, east	$3^h24.4^m$



(6)

East of the meridian, p.m. observation, U.C. in a.m.

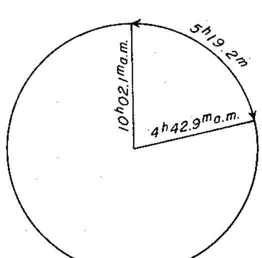
Gr. U.C., Sept. 6, 1972	= $3^h05.2^m$ a.m.
Red. for long.	= −1.3
L.m.t. of U.C., Sept. 6	= $\left\{\begin{array}{l}3\ 03.9\ \text{a.m.}\\+12\end{array}\right.$
L.m.t. of observation, Sept. 5	= 6 34.0 p.m.
Hour angle, east	= $8^h29.9^m$



(7)

East of the meridian, a.m. observation, U.C. in p.m.

Gr. U.C., Mar. 23, 1972	= $1^h59.2^m$ p.m.
Red. for long.	= −1.3
L.m.t. of U.C., Mar. 23	= $\left\{\begin{array}{l}1\ 57.9\ \text{p.m.}\\+12\end{array}\right.$
L.m.t. of observation, same date	= 6 06.6 a.m.
Hour angle, east	= $7^h51.3^m$



(8)

East of the meridian, a.m. observation, U.C. in a.m.

Gr. U.C., May 22, 1972	= $10^h03.4^m$ a.m.
Red. for long.	= −1.3
L.m.t. of U.C., May 22	= 10 02.1 a.m.
L.m.t. of observation, same date	= 4 42.9 a.m.
Hour angle, east	= $5^h19.2^m$

2–43. *Mean time hour angle of Polaris at elongation:* t = the sidereal hour angle in angular measure. This is converted first into the sidereal time interval and then into the mean time interval, which is the mean time hour angle of Polaris at elongation.

$$\cos t = \cot an\ \delta \tan \phi$$

Example of computing the mean time hour angle of Polaris at elongation, April 3, 1970, in latitude 48°00′ N., on which date the declination of Polaris is 89°07′50.3″ N.:

$\phi = 48°00'$; $\tan\phi = 1.11061$
$\delta = 89°07'50.3''$; $\text{cotan}\,\delta = 0.01518$
$\cos t = (0.01518)(1.11061) = 0.01686$
Sidereal hour angle = 89°02′02″
89° = 5^h56^m
2′ = 0^m08^s
2″ = (negligible)
$5^h56^m08^s$
Reduction to mean time hour angle = $-0\ 58$ (sidereal conversion)
Mean time hour angle at elongation = $5^h55^m10^s$

2–44. *Altitude observation of Polaris at upper culmination for latitude:*

$$\phi = h + \delta - 90°$$

Altitude observation of Polaris at lower culmination for latitude: The mean time hour angle of Polaris at lower culmination is 11 hours 58 minutes 2 seconds:

$$\phi = h + 90° - \delta$$

The settings for the approximate altitude of Polaris at upper and lower culminations, respectively, are:

$$v = \phi \pm (90° - \delta)$$

The following program is recommended in altitude observations of Polaris at culmination for latitude.

Compute the local mean time and watch time of culmination.

Thoroughly level the transit.

About four minutes before culmination observe the altitude of Polaris with the telescope in direct position.

Reverse the transit and observe the altitude of Polaris.

Again level the transit.

Observe the altitude of Polaris with the telescope in the reversed position.

Turn the transit to the direct position of the telescope and again observe the altitude of Polaris.

Take a mean observed altitude to use in the reduction.

Example of altitude observation of Polaris at upper culmination for latitude:

September 5, 1972, in approximate latitude 33°23′ N., and longitude 107°11′38″ W., at approximate temperature 50° F., and approximate altitude above sea level 3,600 ft., I make an altitude observation of Polaris at upper culmination for latitude, making four observations, two each with the telescope in direct and reversed positions.

Summary of results

Watch correct for 105th meridian time by comparison with radio signals.
Mean watch time of observation = $3^h16^m31^s$ a.m.
Mean observed vertical angle = 34°16′23″
Reduced latitude = 33°23′22″ N.

Field notation

Setting: 90°00′
$\delta \neq 89°08'$
$90° - \delta \neq 0°52'$
$\phi \neq 33°23'$
$v \neq 34°15' = \phi + (90° - \delta)$

U.C. of Polaris, Gr.m.t., Sept. 5, 1972 = $3^h09.1^m$ a.m.
Red. to long. 107°11.6′ W. (sidereal conversion) = -1.2
$3^h07.9^m$ a.m.

L.m.t. of U.C., Sept. 5, 1972 = $3^h07^m54^s$ a.m.
Correction for longitude (2°11′38″) = +8 47
Computed watch time of U.C. = $3^h16^m41^s$ a.m.

Telescope	Watch time	Vertical angle
Direct	$3^h12^m33^s$	34°15′30″
Reversed	3 14 31	34 17 30
Reversed	3 18 30	34 18 00
Direct	3 20 30	34 14 30
Mean	$3^h16^m31^s$	34°16′23″

Refraction (1′24″ × .89 = 1′15″) = $-1\ 15$
h = 34°15′08″
$\delta = 89°08'14''$; $90° - \delta$ = $-51\ 46$
$\phi = 33°23'22''$ N. = $h - (90° - \delta)$ = 33°23′22″

Hour Angle Observation of Polaris for Latitude

2–45. The latitude may be determined by an altitude observation of Polaris at any hour angle. By this method the vertical angles are read in pairs, or double pairs, with reversals of the position of the telescope, and watch time noted at each setting. A watch correction is required, which will be applied to the mean (or average) of the watch readings to obtain the correct local mean time of observation for the pair or double pair of settings. The mean time hour angle of Polaris at the epoch of observation is then taken out as in observations for azimuth, and the declination of Polaris for the date is ascertained in the Ephemeris.

With the two values, mean time hour angle and declination, the latitude may be computed or there may be derived from the table in the Ephemeris the vertical angle equivalent for the position of Polaris above or below the earth's polar axis at the epoch of observation. The latter value is applied to the observed vertical angle, corrected for refraction, to secure the true elevation of the pole, or the *latitude* of the *station*. The method may be combined with the observation for azimuth, or it may be used independently.

The vertical angle reduction is tabulated in the Ephemeris in a simplified form in which the two principal arguments are employed to secure a primary adjustment to the elevation of the pole, subtractive when Polaris is above the pole and additive below. Since the primary adjustments have been computed for a station in latitude 45° north, a small supplemental correction must then be taken from the table for altitudes other than 45°, the arguments being mean time hour angle and observed altitude.

If an analytical reduction is made, it is convenient to begin with an angle α, computed from the equation:

$\operatorname{Tan} \alpha = \dfrac{\tan \delta}{\cos t}$, in which equation the factor "cos t" becomes negative for hour angles exceeding 90°, whereupon α will exceed 90°. Remember that "t" is the *sidereal* hour angle.

The latitude may then be derived from the equation:

$$\operatorname{Cos}(\phi - \alpha) = \frac{\operatorname{Sin} \alpha \sin h}{\sin \delta}$$

Example of hour angle observation of Polaris for latitude, making use of the table given in the Ephemeris:

June 28, 1972, in approximate latitude 41°20′ N., and longitude 111°37′ W., at approximate temperature 50° F., and elevation above sea level 6,800 ft., I make an hour angle observation of Polaris for latitude, making four observations, two each with the telescope in direct and reversed positions.

Summary of results

Mean observed vertical angle	= 41°55′00″
Mean watch time of observation	= $4^h46^m38^s$ a.m.
Watch fast of local mean time, by comparison with radio time signal corrected for longitude	= 26^m28^s
Reduced latitude	= 41°20′37″ N.

Field notation

Telescope	Vertical angle	Watch time
Direct	41°53′00″	$4^h44^m45^s$ a.m.
Reversed	41 54 00	4 45 50
Reversed	41 56 00	4 47 20
Direct	41 57 00	4 48 37
Mean	41°55′00″	$4^h46^m38^s$ a.m.
Watch fast of local mean time		−26 28

L.m.t. of observation, June 28, 1972 = $4^h20^m10^s$ a.m.
= $4^h20.2^m$ a.m.

Gr. U.C. of Polaris, same date = $7^h38.7^m$ a.m.
Red. to long. 111°37′ W. = −1.2 = $7^h37.5^m$ a.m.

Hour angle of Polaris east of meridian = $3^h17.3^m$

Declination of Polaris 89°08′06″

Mean time hour angle	Primary adjustment, subtractive, Polaris above the pole — Declination 89°08′00″	89°08′06″	89°08′20″
$3^h11.5^m$	0°34′35″	0°34′31″	0°34′22″
3 17.3	0 33 35	0 33 31	0 33 22
3 17.5	0 33 33	0 33 29	0 33 20

Mean observed vertical angle,	v = 41°55′00″
Refraction, 64″ × 0.79	= −0 51
	h = 41°54′09″
Primary adjustment to elevation of pole	= −33 31
Supplemental correction	= −1
Latitude of station	= 41°20′37″ N.

Polaris at Elongation

2–46. Of the various methods of observation to establish the true meridian, the simplest is the observation upon Polaris at eastern or western elongation.

Azimuth of Polaris at elongation:

$$\text{Sin A} = \frac{\cos \delta}{\cos \phi}$$

Example of computing the azimuth of Polaris at elongation, October 20, 1970, in latitude 46°20′ N., on which date the declination of Polaris is 89°07′54″ N.:

$$\cos \delta = 0.015155$$
$$\cos \phi = 0.690462$$
$$\sin A = 0.015155 \div 0.690462 = 0.021949$$

A = Azimuth of Polaris at elongation = 1°15′28″.

A table of azimuths of Polaris at elongation for latitudes from 10° to 70° N. appears in the Ephemeris, arguments: declination of Polaris and latitude of station.

Example in the use of the table of azimuths of Polaris at elongation, same date and station as above, showing the method of interpolation:

Latitude	Declination 89°07′50″	Declination 89°07′54″	Declination 89°08′00″
	Azimuths elongation		
46°00′	1°15′06″	1°15′00″	1°14′52″
46°20′		1°15′28″	
47°00′	1°16′30″	1°16′24″	1°16′15″

By interpolation in the table the required azimuth of Polaris at elongation is therefore found to be 1°15′28″.

Azimuth of Polaris at Any Hour Angle

2–47. While there is no better method for the establishment of the true meridian than the observation upon Polaris at elongation, for most of the year this requires nighttime observations. Moreover, should Polaris be obscured by clouds at the time of elongation, the observation must fail.

The "hour angle" method admits of observa-

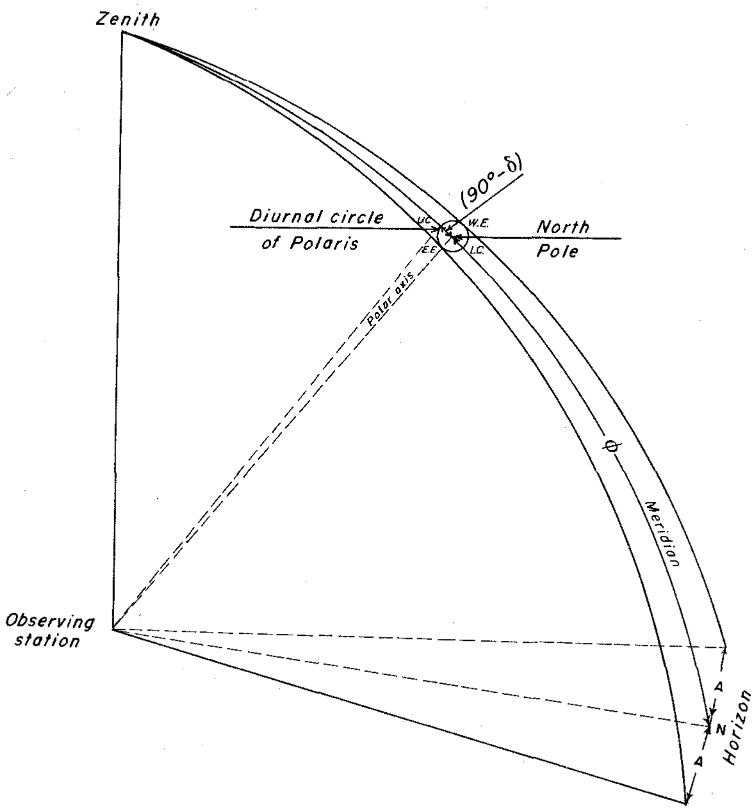


FIGURE 7.—The meridian and vertical planes tangent to the diurnal circle of Polaris as viewed from outside the celestial sphere.

tion upon Polaris for azimuth at any time that the star is visible; the precise watch error in local mean time must be known, but if this has been determined, the hour angle method becomes at once the most convenient. The possible accuracy of the result compares favorably in every way with the refinement obtained in an observation at elongation.

2–48. *Azimuth of Polaris at any hour angle:* t = sidereal hour angle in angular measure; in hour angles exceeding 90° the function "$-\sin\phi\cos t$" becomes positive by virtue of the cosine of an angle between 90° and 270° being treated as negative in analytical reductions:

$$\text{Tan } A = \frac{\sin t}{\cos\phi\tan\delta - \sin\phi\cos t}$$

A table of azimuths of Polaris at all hour angles, for latitudes from 10° to 65° N., appears in the Ephemeris, arguments: declination of Polaris, mean time hour angle, and latitude of station. For other than the latitudes given in the table, and for greater accuracy in terms of seconds of azimuth, the surveyor will be required to solve the above equation.

Example of computing the azimuth of Polaris, February 21, 1972, at a mean time hour angle of $2^h37.4^m$, in latitude 33°20′ N., on which date the declination of Polaris = 89°08′36″ N.:

Mean time hour angle = $2^h37.4^m$	
= $2^h37^m24^s$	$2^h = 30°$
	$37^m = 9°15'$
Red. to sidereal hour angle = 26^s	$50^s = 12'30''$
Sidereal hour angle = $2^h37^m50^s$	= $39°27'30''$
log cos ϕ = 9.921940	log sin ϕ = 9.739975
log tan δ = 1.825278	log cos t = 9.887666
log cos ϕ tan δ = 1.747218	log sin ϕ cos t = 9.627641
nat cos ϕ tan δ = 55.875	nat sin ϕ cos t = 0.424
nat sin ϕ cos t = 0.424(−)	
	log sin t = 9.803127
Algebraic sum = 55.451	log 55.451 = 1.743910
	log tan A = 8.059217

Azimuth of Polaris at above hour angle, A = 0°39′24″

Example in the use of the table of azimuths of Polaris at all hour angles, same date, hour angle, and station as above, showing the method of interpolation:

Mean time hour angle	Azimuth of Polaris — Mean declination + 89°08′30″ — Latitude 32°	33°20′	34°	Correction subtractive for declination + 89°08′40″
$2^h29.6^m$	37.2′	37.8′	38.1′	0.1′
37.4		39.4		0.1
39.6	39.3	39.9	40.2	0.1

By interpolation in the table the required azimuth of Polaris is therefore found to be 0°39.4′ − 0.1′ = 0°39.3′ or 0°39′18″.

Polaris at Sunset or Sunrise

2–49. If the sky is clear Polaris may be most conveniently observed by the hour angle method at sunset or sunrise without artificial illumination. The preparation for the observation consists in computing in advance the approximate settings in azimuth and altitude in order to find Polaris. The plan contemplates an approximate reference meridian. With the time of sunset or sunrise assumed as the time of observation, the hour angle *'t'* and azimuth *"A"* are ascertained in order to find the position of Polaris in azimuth. The vertical angle will be equal to the latitude of the station *plus* the primary adjustment when Polaris is above the pole, or *minus* when below, taking the value from the tabulation given in the Ephemeris.

The "settings" for finding position are approximations, to bring Polaris reasonably near the center of the field of the telescope where the star should be found in plain view. The telescope must be focused upon a distant object, otherwise, though Polaris may be practically at the center of the field, it might be out of focus and therefore may not be noticed during daylight. When Polaris has been found the observation may follow the hour angle method, the reductions to be based upon the data derived in the observation. The settings should be made each time for the several sightings. The daylight hour angle method is particularly desirable because the observation, including all instrumental work, marking of points upon the ground, etc., is accomplished without artificial illumination, and sunset is usually a covenient time to devote to this field duty.

Example of the computation of the position of

Polaris at sunset, May 6, 1972, at a station in latitude 47°20′ N., and longitude 102°40′ W.:

From the Ephemeris the declination of the sun adjusted to approximate sunset is found to be 16°48′ N.; the equation of time 3^m, to be subtracted from apparent time; upper culmination of Polaris, Greenwich meridian $11^h06.2^m$ a.m.; the declination of Polaris + 89°08′16″. From the Standard Field Tables, the apparent time of sunset is found to be 7^h17^m p.m.

May 6, 1972:			
Sunset	=	7^h17^m p.m., app. t.	
Equation of time	=	−3	
Anticipated time of observation	=	7^h14^m p.m., l.m.t.	
		+12	
Gr. U.C. of Polaris = $11^h06.2^m$ a.m.			
Red. to long. 102°40′ = −1.1		11 05 a.m., l.m.t.	
Hour angle of Polaris, west of meridian	≠	8^h09^m	A ≠ 1°03′ W.
Latitude of station	=	47°20′	
Vertical angle adjustment, Polaris below the pole	=	−28	v ≠ 46°52″

Example of the computation of the position of Polaris at sunset, November 6, 1972, same station:

Declination of the sun adjusted to approximate sunset 16°14′ S.; equation of time 16^m to be subtracted from apparent time; upper culmination of Polaris, Greenwich meridian $11^h02.1^m$ p.m.; declination of Polaris + 89°08′35″.

November 6, 1972:			
Sunset	=	4^h46^m p.m., app. t.	
Equation of time	=	−16	
Anticipated time of observation	=	4^h30^m p.m., l.m.t.	
Gr. U.C. of Polaris = $11^h02.1^m$ p.m.			
Red. to long. 102°40′ = −1.1		11 01 p.m., l.m.t.	
Hour angle of Polaris, east of meridian	=	6^h31^m	A ≠ 1°15′ E.
Latitude of station	=	47°20′	
Vertical angle adjustment, Polaris below the pole	=	−08	v ≠ 47°12′

Example of the computation of the position of Polaris at sunrise, November 7, 1972, same station:

Declination of the sun adjusted to approximate sunrise 16°25′ S.; equation of time 16^m to be subtracted from apparent time; upper culmination of Polaris and declination of Polaris same as above.

November 7, 1972:			
Sunrise	=	7^h15^m a.m., app. t.	
Equation of time	=	−16	
Anticipated time of observation	=	6^h59^m a.m., l.m.t.	
		+12	
Upper culmination of Polaris, *November 6*	=	11 01 p.m., l.m.t.	
Hour angle of Polaris, west of meridian	≠	7^h58^m	A ≠ 1°05′ W.
Latitude of station	=	47°20′	
Vertical angle adjustment, Polaris below the pole	=	−26	v ≠ 46°54′

Stellar Observations, Equatorial Belt

2–50. There are two customary methods of star identification. First, the brighter stars may be found individually by naked eye during starlight, each by means of its position within its own constellation and with the aid of a chart that shows the outline of that and the neighboring constellations; second, using the transit, any star may be found by reference to vertical angle and horizontal angle from the meridian, both values related to an anticipated time of observation, and to an approximate north and south line. The second method is frequently more certain, especially if there are clouds that obscure some of the stars and is a necessity for twilight or daylight observations.

The charts of the constellations are interesting and useful, but they are not employed as an accessory to the Manual methods.

The location of any one of the selected bright stars, in favorable position for observation, *on any date and at any moment* within the 24-hour period, may be found most readily by reference to the diagram insert of the Ephemeris; an explanation of its use is given on the diagram. The simple steps are these: first interpolate for the date, and place the meridian line of the overlay scale on the date line; this shows the field as it will be at the noon of that date. Next, move the overlay scale to the left for p.m. periods, or to the right for a.m. periods, as shown by the lower set of figures, to the anticipated time of an observation. Then read the upper set of figures for hour angle for any selected star at that anticipated time, to the east or to the west of the meridian.

Having selected the star to be observed and the anticipated time of observation, the time of the meridian passage of the star for that date is then taken from the Ephemeris. The hour angle for the position is the *time interval* between the anticipated time of observation and the time of the meridian passage.

Use the following equation to find the vertical angle of the star at the anticipated moment of observation:

$$\sin h = \cos t \cos \phi \cos \delta + \sin \phi \sin \delta$$

(If sin h is negative the star is below the horizon.)

Then use the companion equation to find the horizontal angle from the meridian, as follows:

$$\text{Cos A} = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h$$

The product "$\sin \phi \sin \delta$" and the fraction $\dfrac{\sin \delta}{\cos \phi \cos h}$ are negative for south declinations.

The product "$\cos t \cos \phi \cos \delta$" is negative for hour angles exceeding 6 hours or 90°.

If the result for "cos A" is $\left\{ \begin{array}{l} \text{positive} \\ \text{negative} \end{array} \right\}$ the angle counts from the $\left\{ \begin{array}{l} \text{north.} \\ \text{south.} \end{array} \right.$

The vertical angle setting ("v" for this purpose) for the meridian passage of a star south of the zenith is:

$$v \neq 90° - \phi \pm \delta$$

An approximate north and south line, the approximate latitude, and the approximate watch correction to local mean time are used in computing the approximate angular settings for "A" and "v". The objective lens should be set carefully to *celestial focus*. The computed direction will be sufficiently precise to bring the selected star into the field of the telescope at the anticipated time of observation. An explanation of preparing for an observation is also given in the Ephemeris.

For *daylight observations*, and referring especially to stars of first magnitude or brighter that would naturally be selected, the initial or approximate values, and the settings for horizontal and vertical angles, should be such as to bring the star within the middle-third of the field, or roughly within 10′ of the cross-wire intersection.

2–51. A first magnitude star is rated as 1.0; second magnitude 2.0; etc. Brighter than first magnitude is rated, as Capella, 0.2; slightly brighter, as Vega, 0.1; much brighter, as Canopus, -0.9; or still brighter, as Sirius, -1.6; on this scale a magnitude of 2.1 is rated for Polaris. This detail is an important feature of identification.

As an additional aid in star identification, it is helpful to note the positions of the bright planets Venus, Mars, Jupiter, and Saturn. The times of their transits, and their approximate declinations, are tabulated in the Ephemeris for the first and sixteenth day of each month. The planets are "wanderers" (very changeable in position) so that the interpolations between the tabulated dates will be rough, although close enough for identification, and the varying positions will become more readily noted on continued acquaintance. The proximities of the planets to the selected stars, up to about 40 minutes in time of transit and 10° difference in declination, are shown in the tabulations by footnote-reference.

The planets appear different in the telescope, Venus very bright and slightly crescent when farthest from the sun, and not so bright, but decidedly crescent when near the sun. Mars is always a dull red, brightest when its transit is near midnight, but scarcely noticeable when the transit is within two or three hours of noon. Jupiter, when passing the meridian between 9 p.m. and 3 a.m., is very bright with some of its several moons always in evidence clearly seen on sharp focus. Saturn, similarly, though not so bright, has its "rings," but no moons. As in the case of Mars, Jupiter and Saturn are scarcely noticeable when their meridian passage occurs between 9 a.m. and 3 p.m., their positions then being so far distant from the earth.

If a star identification is to be made naked-eye from the constellations, then the nearby planet or planets should be noted, and accounted for by appearance. If the star is identified instrumentally, by settings in horizontal and vertical angle, then the noting of the relation to the nearby planet, or planets, is one of the best ways in which to become acquainted with the latter.

2–52. The stellar observation is useful any time of year, particularly when the sun reaches a meridian altitude exceeding 60° or 65°.

There is no difficulty in picking up the meridian passage of a star when the conditions for visibility are good. Most of the selected stars are brighter than Polaris; some of them can be observed throughout any day that is clear and free from haze.

After the initial preparations have been made for a Polaris observation, including the marking of a meridian reference by solar transit orientation, or by reference to lines previously determined, it is good practice to include the meridian passage of a star in the observing program. In this manner the watch correction for local mean time is obtained just when needed and on the most direct plan under the usual field conditions.

The Greenwich mean times of the meridian transit of the selected bright stars of the equatorial belt are tabulated in the Ephemeris for the 1st and 16th day each month; the reductions to the other days of the month are indicated on each page of the stellar tabulations. This data must be converted to the local mean time of transit.

Example of the computation of the finding positions of an equatorial star:

March 10, 1971, on a meridian previously established, at a station in latitude 42°15′ N., and longitude 121°47′ W., preparing to observe the star No. 15/26, α Leonis (Regulus), declination + 12°06′24″, Mag. 1.3, anticipated time of observation 6^h30^m p.m., l.m.t.

Star's transit, Gr.m.t., March 1	$11^h30.4^m$ p.m.
Reduction to March 10	−35.4
Sidereal conversion, long. 121°47′ W.	−1.3
Star's transit, l.m.t.	$10^h53.7^m$ p.m.
Anticipated time of observation, l.m.t.	6 30 p.m.
Hour angle SE. (t = 65°55′)	$4^h23.7^m$

For the purpose of computing the *finding* positions, the mean time hour angle need not be converted to a sidereal time hour angle, since the actual time of the observation will probably vary somewhat from the anticipated time.

$$\sin h = \cos t \cos \phi \cos \delta + \sin \phi \sin \delta$$
$$\cos t = 0.4081$$
$$\cos \phi = 0.7402 \qquad \sin \phi = 0.6724$$
$$\cos \delta = 0.9778 \qquad \sin \delta = 0.2097$$
$$\text{Product} = 0.2954 \qquad \text{Product} = 0.1410$$
$$\sin h = 0.2954 + 0.1410 = 0.4364$$
$$h = \underline{\underline{25°53'}}$$

$$\cos A = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h$$
$$\sin \delta = 0.2097$$
$$\cos \phi = 0.7402 \qquad \tan \phi = 0.9083$$
$$\cos h = 0.8997 \qquad \tan h = 0.4852$$
$$\frac{\sin \delta}{\cos \phi \cos h} = 0.3149 \qquad \tan \phi \tan h = 0.4407$$
$$-0.4407$$
$$\cos A = -0.1258 \text{ (Negative indicates horizontal angle counts from south.)}$$
$$A = \underline{\underline{\text{S. } 82°46' \text{ E.}}}$$

Using the finding positions of the star (h = 25°53′, A = S. 82°46′ E.), four observations are made, two each with the telescope in direct and reversed positions. The *true* values for time and azimuth are obtained by a more precise reduction of the observations by use of the above formulas. Keep in mind that in all stellar observations the true vertical angle (h) is equal to the observed vertical angle (v) minus the refraction (r) in zenith distance. There is no correction for parallax. In solving for time, the hour angle (t) obtained by use of the formula is in terms of the *sidereal* rate. A subtraction of 10 seconds per hour (sidereal conversion) will give the equivalent mean time hour angle.

Direct Solar Observations

2–53. *Declination.* The declination of the sun is corrected in hourly difference to the longitude of the station and to the time of observation. North declinations are treated as positive, south declinations as negative. A northerly hourly motion is treated as positive, a southerly hourly motion is treated as negative.

Example of computation of the sun's declination:

It is desired to compute the value of the sun's declination at a station in longitude 5^h08^m west of Greenwich; apparent time of observation 3^h42^m p.m.; Mar. 18, 1970:

Declination of the sun at Greenwich apparent noon = 1°00′04″ S.

Difference in time from Greenwich apparent noon to apparent time of observation:

For longitude 5^h08^m
For time, p.m. = $+3^h42^m$
$8.83^h = 8^h50^m$

Hourly difference in declination = +59.28″

Difference in declination from Greenwich apparent noon to apparent time of observation:

8.83 × 59.28″ = 523″	= 8′43″ N.
True declination of the sun	0°51′21″ S.

2–54. *Meridian Observation of the Sun for Apparent Noon.* With the telescope on the meridian elevated to the sun's altitude, the watch times of transit of the sun's west and east limbs are noted, the mean of which is the watch time of apparent noon. If the observation fails for either limb the reduction to the sun's center is accomplished by adding or subtracting 68 seconds; a refinement in the amount of this interval is had by referring to the Ephemeris for the time of the sun's semi-diameter passing the meridian for the date of observation.

The setting for the approximate altitude of the sun's center is:

$$v \neq 90° - \phi \pm \delta$$

OBSERVING PROGRAM

Determine the meridian by the best means at hand and compute the vertical angle setting for the sun.

Level the transit, align the instrument on the meridian, and elevate the telescope to the altitude of the sun's center.

Note the watch time of the sun's west limb tangent to the vertical wire.

Note the watch time of the sun's east limb tangent to the vertical wire.

Take the mean of the readings for the watch time of apparent noon from which to compute the watch error local mean time.

The refinement in this observation depends mostly on the direction of the sighting for meridian. A small discrepancy in direction is scarcely appreciable for ordinary requirements, such as to establish a watch correction in local mean time with necessary accuracy for making the Polaris observations for azimuth and latitude by the hour angle method.

Example of meridian observation of the sun for apparent noon, and reduction to watch correction local mean time:

September 9, 1971, in latitude 42°32′24″ N., and longitude 119°46′30″ W. ($7^h59^m06^s$), with the telescope in the meridian and elevated to the sun's altitude, I observe the sun's transit for time, noting the watch time of transit of each limb.

Setting:	90°00′	
$\phi \neq$	(−)42°32′	
$\delta \neq$	(+) 5°20′	(corrected for longitude)
$v \neq$	52°48′	

Watch time of transit, W. limb	=	$11^h54^m48^s$
Watch time of transit, E. limb	=	11 56 56
Watch time of apparent noon	=	$11^h55^m52^s$
Apparent noon = $12^h00^m00^s$		
Eq. of time adjusted to time of observation = −2 37		
Local mean time of apparent noon	=	11 57 23
Watch slow of local mean time	=	1^m31^s

2–55. *Meridian Altitude Observation of the Sun for Latitude.*—Reverse the sign of δ for south declinations:

$$\phi = 90° + \delta - h$$

The following observing program is recommended:

Thoroughly level the transit and place the telescope in the meridian elevated to the sun's approximate altitude at noon.

Observe the altitude of the sun's lower limb with the sun slightly east of the meridian.

Reverse the transit.

Observe the altitude of the sun's upper limb with the sun slightly west of the meridian.

Take the mean observed vertical angle for the altitude of the sun's center at apparent noon.

The important factor in this observation is exactness in vertical angle. The observation may be duplicated by vertical angle readings on stars within the equatorial belt at meridian passage, using the same equation:

$$\phi = 90° \pm \delta - h$$

The resulting values in latitude should agree within the limits of the precision of the instrument. The uncertain factor is the value of the observed vertical angle. This may be compensated by balancing an observation within the equatorial belt by an observation on Polaris at upper or lower culmination, or by a latitude observation on Polaris by the hour angle method.

Example of meridian altitude observation of the sun for latitude:

September 25, 1971, in approximate latitude 48°10′ N., and longitude 109°10′ W. ($7^h16^m40^s$), temperature 70° F., elevation above sea level 2,500 ft., I make an observation of the sun for latitude, observing the altitude of the sun's lower limb with the telescope in direct position, reversing the telescope and observing the sun's upper limb.

Settings:

	90°00′
$\phi \neq$	(−)48°10′ N.
$\delta \neq$	(−) 0°49′ S.
$v \neq$	41°01′

Sun's semi-diameter $\neq$ 16′

Lower limb (41°01′ − 16′)	40°45′
Upper limb (41°01′ + 16′)	41°17′

$\underline{\odot}$ Observed alt., lower limb, telescope dir.	= 40°46′30″
$\overline{\odot}$ Observed alt., upper limb, telescope rev.	= 41°18′30″
Mean observed altitude, v	= 41°02′30″
Refraction	= −0 59
Parallax	= +0 06
h	= 41°01′37″

Declination, Gr. app. noon = 0°42′01.0″ S.
Red. to longitude 109°10′ W., 7.278 × 58.45″ = 7′05.4″ S.
0°49′06.4″ S.

δ = 0°49′06″ S.

$\phi = 90° - \delta - h$ = 48°09′17″ N.

Example of meridian observation of the sun for time and latitude:

September 10, 1969, in approximate latitude 41°35′ N., and longitude 109°58′ W., at temperature 50° F., and elevation above sea level 6,500 ft., I make a meridian observation of the sun for time and latitude, observing simultaneously the altitude of the sun's lower limb and the transit of the sun's west limb, reversing the telescope and observing simultaneously the altitude of the sun's upper limb and the transit of the sun's east limb.

Setting:

	90°00′
$\phi \neq$	(−)41°35′ N.
$\delta \neq$	(+) 4°47′ N.
$v \neq$	53°12′

Sun's semi-diameter $\neq$ 16′

Lower limb (53°12′ − 16′)	52°56′
Upper limb (53°12′ + 16′)	53°28′

Telescope	Sun	Watch time transit	Observed vertical angle
Direct	$\underline{\odot}$	$11^h56^m18^s$	52°56′30″
Reversed	$\overline{\odot}$	11 58 26	53 29 00
Mean		$11^h57^m22^s$	53°12′45″
Refraction (43″ × 0.8)		=	−34
Parallax		=	+05
h		=	53°12′16″
δ = 4°47′18″ N.; 90° + δ		=	94°47′18″
$\phi = 90° + \delta - h$		=	41°35′02″

Watch time of apparent noon		= $11^h57^m22^s$
Apparent noon	= $12^h00^m00^s$	
Equation of time, Gr. noon subtractive from app. t	= 3^m01^s	
Red. to long. 109°58′ W.	= 7	
	3^m08^s	−3 08
Local mean time of apparent noon	$11^h56^m52^s$	= 11 56 52
Watch fast of local mean time		= 30^s

The accuracy of the reduced latitude is directly related to the refinement of the value of the observed vertical angle. A better determination of the latitude by this method is possible only by making a series of observations on successive days, or the observation may be duplicated by vertical angle readings on stars within the equatorial belt at meridian passage, and by combining the result with Polaris observations for latitude.

Altitude Observation of the Sun for Azimuth

2–56. While observations of Polaris for azimuth are used extensively, there are situations where a direct altitude observation on the sun will expedite the survey. The bright stars within the equatorial belt may also be substituted when the sun is not in favorable position. In general these observations are supplemental to the normal running of the lines by solar transit orientation in order to improve and verify the line work.

The altitude observation will frequently permit a prompt start on the survey in advance of an opportunity for the usual Polaris observation. Also, at stations far removed from field headquarters, the solar transit orientation should be verified in this manner. Direct observations on the lines as run will help to show that the solar unit is performing well or that it needs adjustment.

Some of the stars are always in favorable position, both in hour angle and declination. The brighter stars may be picked up during the daylight hours if not obscured by haze or clouds. The stellar altitude observations are particularly useful when the sun is not in favorable position. In the southern States, during the summer months, the sun is too high for the noon observation for time and latitude. In the northern States, from late October until late February, the sun is too low for the best observing for time and azimuth. The bright north declination stars are especially helpful for the observing in Alaska, the southern declination stars for the meridian time-and-latitude observations in Florida.

The trigonometric elements of the altitude observation for time and azimuth are vertical angle, latitude, and declination of the sun or the star.

Accuracy in latitude is essential to good observing for azimuth by the altitude method. If the latitude has not been well determined previously, an azimuth observation on the sun southeasterly should be balanced by one southwesterly at about the same vertical angle. Averaging the results will eliminate the effect of an unknown discrepancy in latitude.

The precision with which the azimuth may be determined by the altitude observation of the sun or a star is dependent on the correctness of the vertical angle. The error in azimuth that results from a discrepancy in vertical angle increases rapidly when the angular elevation is large, when the hour angle is small, or with the southerly declinations. Presuming careful observing with the instrument in good adjustment, but with an error in the readings of vertical angles, the error in azimuth is multiplied one, two, or three times, depending on the sun's position in altitude, hour angle, and declination. The effect is shown graphically in figure 8. By balancing an observation southeasterly with one southwesterly at about the same vertical angle, the error in azimuth will be compensated.

The altitude observation calls for accuracy in the instrumental adjustments and for good judgment in the selection of a well formed pole-zenith-sun triangle. Vertical angles from 20° to 50° are to be preferred, not less than three hours from meridian passage, and north declination. A bright star in north declination is much better than the sun when the south declination of the sun exceeds 10°.

In order to balance the altitude observation for azimuth, to compensate for uncertainties in vertical angle, the sun may be observed *southeasterly and southwesterly;* or the sun in one position and a star in the companion position; or two north declination stars may be selected, especially when the sun is in southerly declination; etc.; the purpose being to balance the observation in nearly the same vertical angle, and to secure well-shaped celestial-triangles.

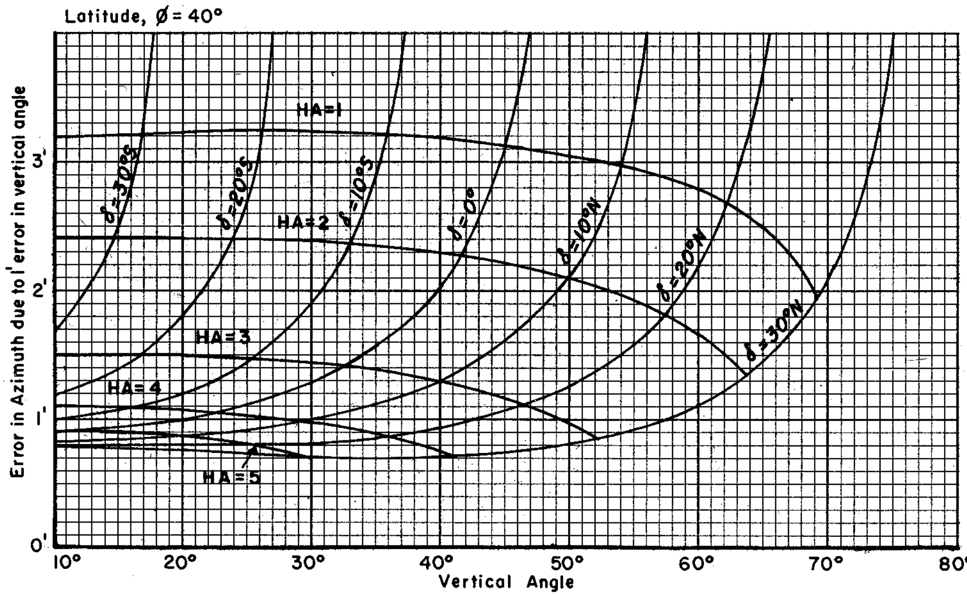


FIGURE 8.—Errors in azimuth caused by one minute error in vertical angle at various angular elevations, computed for observations in latitude 40°, for declinations within the equatorial belt.

The solar transit is equipped with a full vertical circle, a neutral-tint colored glass in the dust shutter of the eyepiece, a removable prismatic eyepiece, and a removable reflector for illuminating the cross wires. These are essential to rapid and accurate altitude observations, and for the night observing. The latest model features a solar circle on the reticle of the transit telescope; this gives the horizontal and vertical angle sightings to the sun's center (instead of to the limbs). Double lines are provided for half of each the vertical and horizontal cross wires; this spacing is to improve the *stellar* observation for exact centering, avoiding the complete covering of the star by the wire (as the latter may obscure the star in the daylight observation). See section 2-64.

There are a number of equations for solving the altitude observation for *azimuth,* in which the elements are vertical angle, latitude, and declination of the sun or the star. These are companion equations to those employed in solving the altitude observation for *time,* using the same elements. Some of the equations are adapted to the use of natural trigonometric functions and the computing machine; the same equations may be employed by logarithmic reduction in combination with the natural functions; some are adapted to strictly logarithmic reduction. These equations are given in the Standard Field Tables. Occasionally, for a check against a possible error, if the results do not come out as expected, a second reduction may be made, using another equation.

The same equations are employed for the stellar observations as for the altitude observation of the sun. With the stars there is no reduction to center, no correction for parallax, and no hourly change in declination. The sun and the stars have the same corrections for *refraction in zenith distance,* the latter subject to temperature change and to differences in barometric pressure.

Under the Manual rules, a series of three altitude observations upon the sun, each with the telescope in direct and reversed position, are required. Each pair of direct and reversed sightings are combined and reduced as one observation. This will give three results for the indicated bearing of the reference mark. The separate results will vary somewhat, much the same as separate orientations of the solar unit. When desired, in order to guard against error, or to check a discrepancy, any of the sightings may be reduced to the sun's center and solved separately.

For the *stellar* altitude observation four sightings are required, two each with the telescope in direct and reversed position, to be reduced as one observation. The number of the observations may be increased if desirable, although it is good practice to limit the number of sightings to not over six in any one series. Any of them may be reduced separately if desired to check against an error in the reading of the angles. As each sighting is "centered" on the star, the differences in the *rate of travel* in time, horizontal angle, and vertical angle, will be uniform.

It is emphasized that none of the reduced altitude observations for azimuth, in terms of the indicated bearing of the reference mark, standing alone as one observation, can be regarded as within the attainable limit of accuracy of the one-minute transit until duly verified by a completely independent method, such as the Polaris observation to check the altitude observation, or the altitude observation southeasterly balanced with one southwesterly.

2–57. An altitude observation of the sun for azimuth consists in the simultaneous determination of the true vertical and horizontal angles to the sun's center, the horizontal angle being referred to a fixed point. With the true vertical angle to the sun's center, the declination of the sun, and the latitude of the station all known, one of the following equations is entered and a calculation made of the azimuth of the sun's center at the epoch of observation, as referred to the true meridian; the relation between the sun's calculated azimuth and the recorded angle to the sun's center gives the bearing of the reference point.

Altitude observation of the sun for azimuth, *first formula*—Reverse the signs of "δ" for south declinations:

$$\text{Tan } \tfrac{1}{2} A = \sqrt{\frac{\cos \tfrac{1}{2} (\zeta+\phi+\delta) \, \sin \tfrac{1}{2} (\zeta+\phi-\delta)}{\cos \tfrac{1}{2} (\zeta-\phi-\delta) \, \sin \tfrac{1}{2} (\zeta-\phi+\delta)}}$$

The spherical angles "ζ", "ϕ", and "δ" appear in this equation combined as in one formula for the reduction of an altitude observation of the sun for apparent time.

2–58. Altitude observation of the sun for azimuth, *second formula*—For south declinations the function "sin δ" becomes negative by virtue of the sine of a negative angle being treated as negative in analytical reductions: If the algebraic sign of the result is positive the azimuth "A" is referred to the north point, but if negative, the azimuth "A" is referred to the south point:

$$\text{Cos } A = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h$$

2–59. Altitude observation of the sun for azimuth, *third formula*—The following equation is expressed directly in terms of the spherical triangle "pole-zenith-sun:" Reverse the sign of "δ" for south declinations:

Pole to zenith $= 90° - \phi =$ colat.;
Pole to sun $= 90° - \delta =$ codecl.;
Zenith to sun $= 90° - h =$ coalt.;
$S = \frac{1}{2}$ sum of the three sides:

$$\text{Cos } \tfrac{1}{2} A = \sqrt{\frac{\sin S \sin (S - \text{codecl.})}{\sin \text{colat.} \sin \text{coalt.}}}$$

Observing Program, Morning

2–60. Thoroughly level the transit.

With the telescope in direct position observe and record the horizontal angle from a fixed reference point to the sun's right limb, and the vertical angle to the sun's upper limb; these observations must be simultaneous; the sun will appear as indicated; note the watch time at the moment of the observation:

Reverse the transit.

Observe and record the horizontal angle from the fixed reference point to the sun's left limb, and the vertical angle to the sun's lower limb; these observations must be simultaneous; the sun will appear as indicated; note the watch at the moment of the observation:

The mean observed vertical and horizontal angles, and the mean watch time are to be used in the reduction; this constitutes one observation, which is repeated until a series of three direct and reversed sightings are made.

Observing Program, Afternoon

2–61. In the afternoon the program is modified only as to the order in which the sun's limbs are observed, which is as follows:

First observation, telescope direct, observe the sun's right and lower limbs:

Second observation, telescope reversed, observe the sun's left and upper limbs:

2–62. By the above observing programs the horizontal and vertical angles in the direct positions of the telescope will be found of about the same numerical values as in the reversed position of the telescope, by reason of the sun passing in a direction that will carry it across the field of the telescope during the time taken in the reversal and second setting. Differential refraction is therefore eliminated; it is desirable that the corresponding angles in the direct and reversed positions of the telescope be about the same rather than as far apart as would result in any other observing plan.

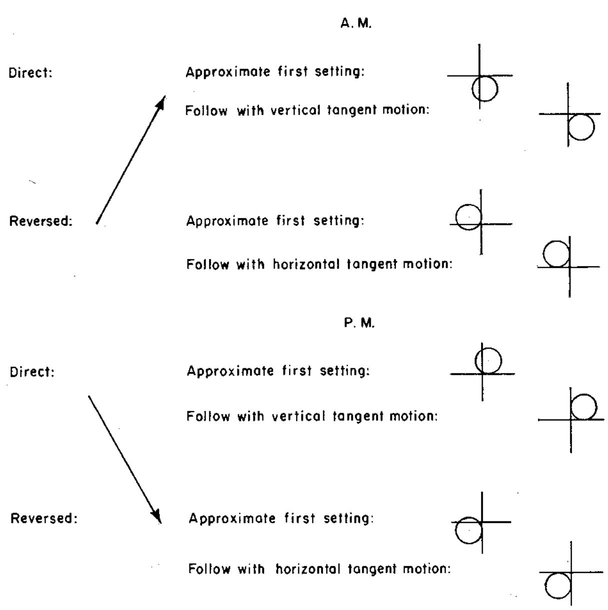
The most suitable hour for this observation is when the sun is moving rapidly in altitude. When the sun has been brought into about the proper position in the field of the telescope, the observer by horizontal tangent motion on the plates keeps the vertical wire tangent to the sun's right or left limb while the upper or lower limb of the sun by the direction of its motion gradually approaches the horizontal wire; at the moment of proper tangency of the two limbs to the two wires the observation is completed by calling "time," stopping all motion until the angles are recorded. It is very helpful for an assistant to read the time and to enter all records.

The data for each altitude observation, resolved to the sun's center, are obtained with minimum involvement through the steps that have been outlined in the observing plan described above. This is recommended until skill in the technique of the observing has been acquired. After that has been accomplished, the period that is required for the observing may be shortened by arranging the recording on the plan show below, the six sightings to be reduced as one observation. The check against chance error in the readings is secured through comparing the means A-B-C that are indicated, which should be about the same numerically

	Tel.	This order, A. M. ↗	This order, P. M. ↘	Watch time	Hor. Ang.	Vert. Ang.
1	Dir.	Upper-right	Lower-right			
6	Rev.	Lower-left	Upper-left			
			Mean	A	A	A
2	Dir.	Upper-right	Lower-right			
5	Rev.	Lower-left	Upper-left			
			Mean	B	B	B
3	Dir.	Upper-right	Lower-right			
4	Rev.	Lower-left	Upper-left			
			Mean	C	C	C
			Mean of all	A–B–C	A–B–C	A–B–C

provided the time spacing is nearly uniform from 1 to 2, 2 to 3, 4 to 5, and 5 to 6. Any large discrepancy in the means will indicate a misreading at some point. If the means are slightly irregular, the *differences* from 1 to 2, 2 to 3, 4 to 5, and 5 to 6, which should be proportional, may be checked by slide rule method.

An equivalent observing plan, thought by many surveyors to be a simpler tangent-motion manipulation, may be substituted if desired, as follows:



2–63. Example of direct altitude observation of the sun for azimuth and time, sun north declination:

The altitude observations are made of the sun, each with the telescope in direct and reversed positions, observing opposite limbs of the sun. The horizontal angle is read from a flag on line to the east, southward to the sun. The known position of the instrument station is in latitude 41°22′40″ N., and longitude 111°46′40″ W. Observation is begun at 9:15 a.m., l. m. t., with watch set to approximate local mean time.

The declination of the sun for the mean period of the three observations is 17°09′30″ N.

The following reductions are made to obtain the true vertical angles of the above observations:

Observation	Telescope	Sun	Watch time			Vertical angle			Horizontal angle, flag to sun		
1st set	Direct	☉	9^h	15^m	05^s	46°	34′	00″	21°	00′	00″
	Reversed	☉	9	15	59	46	10	00	20	29	00
	Mean		9^h	15^m	32^s	46°	22′	00″	20°	44′	30″
2d set	Direct	☉	9^h	17^m	02^s	46°	54′	00″	21°	28′	00″
	Reversed	☉	9	17	36	46	26	00	20	49	00
	Mean		9^h	17^m	19^s	46°	40′	00″	21°	08′	30″
3d set	Direct	☉	9^h	18^m	41^s	47°	12′	00″	21°	52′	00″
	Reversed	☉	9	19	20	46	45	00	21	16	00
	Mean		9^h	19^m	00^s	46°	58′	30″	21°	34′	00″

By 1st obsn. flag bears .... N. 89°59′10″ E.
By 2nd obsn. flag bears .... N. 89 59 23 E.
By 3rd obsn. flag bears .... N. 89 59 08 E.

Mean true bearing of flag .... N. 89°59′14″ E.
Watch slow of l. m. t., 1st obsn. .... = 25^s
" " " " 2nd " .... = 20
" " " " 3rd " .... = 25

Mean watch time slow of l. m. t. .... = 23^s

	1st obsn.	2nd obsn.	3rd obsn.
v	46°22′00″	46°40′00″	46°58′30″
Refraction	−55″	−55″	−54″
Parallax	+06″	+06″	+06″
h	46°21′11″	46°39′11″	46°57′42″

The reductions by formula for azimuth and time are usually made by use of natural functions and calculators. Logarithms have been employed here as a sometimes useful substitute method.

The following examples of reduction are all by the equation:

$$\cos A = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h$$

1st set:

log cos ϕ	9.875274	log sin δ	9.469842	log tan ϕ	9.944941
log cos h	9.838983			log tan h	0.020520
log	9.714257	log	9.714257	log	9.965461
		log	9.755585	nat−	.92355
		nat+	.56962	nat+	.56962
				Cos A−	.35393

True bearing of sun S.69°16′20″ E.
Angle, flag to sun +20°44′30″

S.90°00′50″ E.
True bearing of flag N.89°59′10″ E.

2d set:

log cos ϕ	9.875274	log sin δ	9.469842	log tan ϕ	9.944941
log cos h	9.836586			log tan h	0.025074
log	9.711860	log	9.711860	log	9.970015
		log	9.757982	nat−	.93328
		nat+	.57277	nat+	.57277
				Cos A	.36051

True bearing of sun	S.68°52′07″ E.
Angle, flag to sun	+21°08′30″
	S.90°00′37″ E.
True bearing of flag	N.89°59′23″ E.

3d set:

log cos ϕ	9.875274	log sin δ	9.469842	log tan ϕ	9.944941
log cos h	9.834095			log tan h	0.029762
log	9.709369	log	9.709369	log	9.974703
		log	9.760473	nat−	.94342
		nat+	.57607	nat+	.57607
				Cos A−	.36735

True bearing of sun	S.68°26′52″ E.
Angle, flag to sun	+21°34′00″
	S.90°00′52″ E.
True bearing of flag	N.89°59′08″ E.

The above observations are reduced for time by the equation:

$$\text{Cos } t = \frac{\sin h}{\text{Cos } \phi \cos \delta} - \tan \phi \tan \delta$$

1st obsn.:

log cos ϕ	= 9.875274	log sin h	= 9.859503	log tan ϕ	= 9.944941
log cos δ	= 9.980228			log tan δ	= 9.489614
	9.855502		9.855502	log	= 9.434455
		log	= 0.004001	nat(−)	= .27199
		nat (+)	= 1.00925		
			.27199		
		cos t	= .73726		

t = 42°30′05″ = $2^h50^m00^s$	= $9^h10^m00^s$	a. m.
Equation of time	= +5 57	
L.m.t. of observation	= $9^h15^m57^s$	a. m.
Watch time of observation	= 9 15 32	a. m.
Watch slow of l.m.t.	= 25^s	

2d obsn:

log sin h	= 9.861660
log cos ϕ cos δ	= 9.855502
log	= 0.006158
nat (+)	= 1.01428
nat tan ϕ tan δ (−)	= .27199
cos t	= .74229

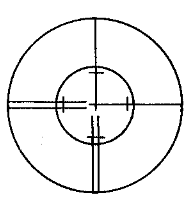
t = 42°04′23″ = $2^h48^m18^s$	= $9^h11^m42^s$	a. m.
Equation of time	= +5 57	
L.m.t. of observation	= $9^h17^m39^s$	a. m.
Watch time of observation	= 9 17 19	a. m.
Watch slow of l.m.t.	= 20^s	

3d obsn:

$\log \sin h$	$= 9.863856$
$\log \cos \phi \cos \delta$	$= 9.855502$
$\log$	$= 0.008354$
nat $(+)$	$= 1.01942$
nat $\tan \phi \tan \delta$ $(-)$	$= .27199$
$\cos t$	$= .74743$
$t = 41°37'54'' = 2^h46^m32^s$	$= 9^h13^m28^s$ a. m.
Equation of time	$= +5\ 57$
L.m.t. of observation	$= 9^h19^m25^s$ a. m.
Watch time of observation	$= 9\ 19\ 00$ a. m.
Watch slow of local mean time	$= 25^s$

The Solar Circle

2–64. The design of the reticle of the transit telescope to include a circle that is equal to the image of the sun's diameter adds a desirable improvement to the technique of the altitude observation for azimuth. There are two advantages, first, all sightings for vertical angle and horizontal angle read to the sun's center; second, the manipulation of the vertical and horizontal tangent-motions to the position of concentric fitting of the circle to the sun's image may be accomplished with utmost certainty that the values for the vertical and horizontal angles are exactly simultaneous. Any single sighting may be reduced separately, if desired; or, in the event of a suspected misreading of an angle, the *differences* between the several sightings, in travel time, vertical angle, and horizontal angle, which should be proportional, may be quickly checked to make certain which reading, if any, shows a discrepancy in excess of what should be expected in good observing.



The solar circle has a radius of 15′45″; this is spaced for the sun's semidiameter on July 1, which is the approximate minimum for the year.

The design of the reticle provides for stadia observations by using both the vertical and horizontal rod, on the ratio of 1:132.

The double cross wires in the left and in the lower halves (direct position of the telescope) are spaced at 40″; this is to improve the daylight stellar observation. The double lines avoid the covering of the star by the cross wire (which may easily obscure the star). The centering and the manipulation of the tangent motions is indicated in the following diagrams:

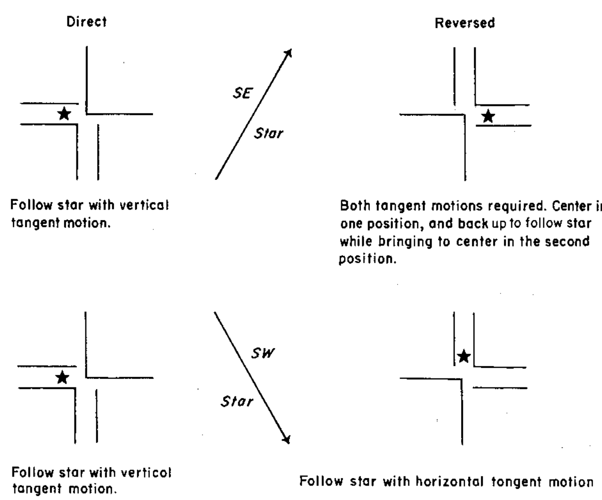


FIGURE 9.—Tangent motion in daylight stellar observation.

Example of notation in direct altitude observation of the sun for azimuth, using a transit equipped with the solar circle:

Observation	Telescope	Apparent time	Vertical angle	Horizontal angle from reference to sun
1	Direct	$8^h13^m25^s$	39°57′00″	34°38′00″
2	Direct		40°02′00″	34°33′00″
3	Direct		40°07′00″	34°29′00″
4	Reversed		40°20′30″	34°18′00″
5	Reversed		40°25′30″	34°14′00″
6	Reversed	8 16 25	40°32′00″	34°09′00″
Mean		$8^h14^m55^s$		

Equal Altitude Observations of the Sun for Meridian

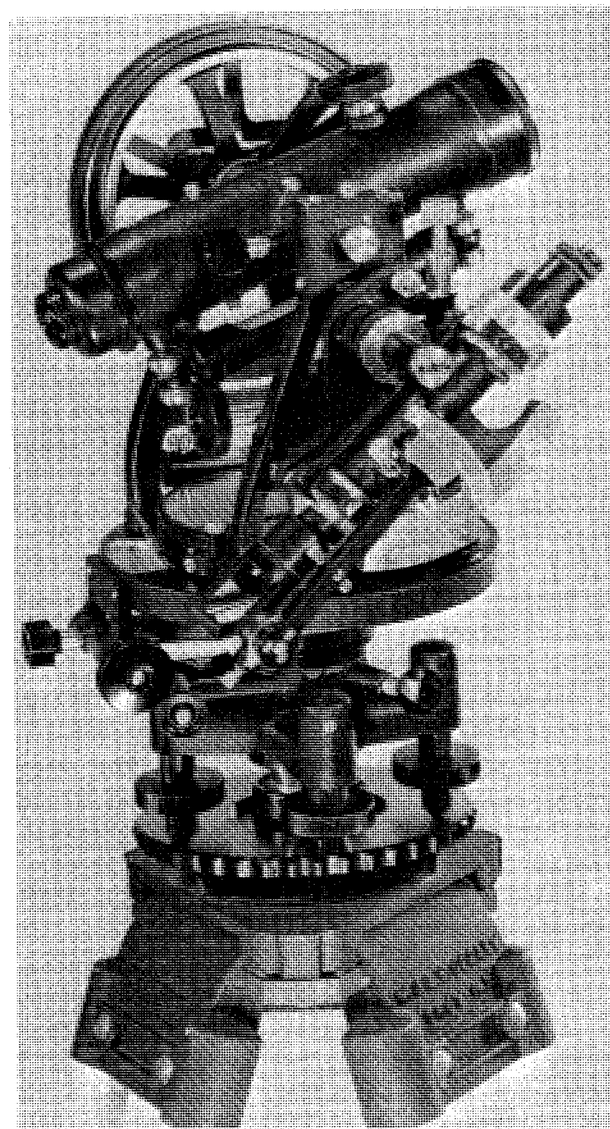
2–65. The true meridian may be established by the method of equal altitude observations of the sun. The observation is not well adapted to line work, but it possesses a certain usefulness in camp, in that the surveyor may thus determine the true meridian by the sun with mere approximations as to time and latitude.

The fixation of the true meridian by this method depends upon the theory that the sun's center at equal altitudes occupies symmetrical positions in azimuth east and west of the meridian in the morning and in the afternoon except for the correction necessary to be applied due to the change in the sun's declination in the interval between the a.m. and p.m. observations. The formula for this correction appears in the Standard Field Tables.

The symmetry of the equal altitude observation is maintained by observing opposite limbs in azimuth in the a.m. and p.m. observations, in connection with the same limb in vertical angle in both observations.

The Solar Transit

2–66. Beginning with the Burt solar compass in 1836, a number of instruments have been designed to solve mechanically the pole-zenith-sun celestial triangle by means of an attached solar unit. Because such instruments can be oriented rapidly without reference to a backsight or a new direct observation, they are suited to surveying through timber, dense undergrowth, and mountainous terrain. The modern solar transit is fully equipped for making the necessary stellar and solar observations by direct means as well. A standard model of the solar transit is shown in Plate No. 1.



Standard model of the solar transit.

The solar unit is in fact a second instrument, operating in its own right independently. Latitude and declination arcs remain clamped and set to their proper values. The polar axis conforms with the line of collimation of the solar telescope. The mounting is designed to bring the vertical plane of the polar axis into parallel with the vertical plane of the transit.

When oriented, the vertical plane of the polar axis is in the plane of the great circle of the meridian. When turned in hour angle at the moment of an observation, the plane normal to the axis of the reflector is in the plane of the great circle that passes through the pole and the sun. The sun's hour angle at that moment is the angle measured along the plane of the Equator, intercepted between the plane of the meridian and the plane of the great circle that passes through the pole and the sun. This angle reads "apparent time" on the hour circle of the solar unit.

The vertical angle inclination of the polar axis equals the latitude of the station; this angle is set on the latitude arc. The angle on the plane of the great circle that passes through the pole and the sun, counting between them equals 90° minus the sun's north declination, or 90° plus the sun's south declination, corrected by an increment equivalent to the refraction in polar distance. The settings for this angle are computed for each day in advance; it is set on the declination arc to agree with the apparent time of observation. The correct position of the sun's zenith distance measured on the vertical plane of the great circle that passes through the sun is secured by the careful leveling of the transit.

After setup and careful leveling, the solar transit may be instrumentally oriented by an experienced surveyor in less than two minutes. The accuracy or acceptable "tolerance" is equal to that of any single, unverified, average direct altitude observation on the sun.

In line running, through timber and undergrowth, there may be 20, 30, or more setups to the mile, each by solar orientation without cutting or opening the line to secure an exact backsight. In this practice, the net result for the mile is the mean of the whole number of the observations, in which many of the smaller differences are compensated, and in which the azimuth of the line between the monuments should normally be brought well inside of the tolerance of 1′ 30″.

What is more, each azimuth determination gives the angular value referred to the true north at that station. This of course is the only method by which a true parallel of latitude can be run by instrumental orientation. The determinations of the true parallel by the "tangent" or "secant" method require the careful running of a "back-and-foresight" line with measured offsets.

Use of the solar unit avoids the cumulative error normally encountered in long "back-and-foresight" lines and in traverse lines where there are many turns. A traverse line may be run *by occupying each alternate station,* cutting in half the time required for the instrumental work. Heavy winds or insecure ground, windfalls, timber, undergrowth, and obstructions that require offset are not in themselves any preventative to rapid and accurate solar orientation.

2–67. The instrumental orientation of the solar unit is made possible through five elements in the construction, as follows:

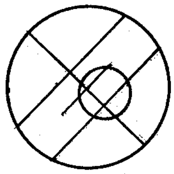
(1) A telescope whose line of collimation is the polar axis; the polar axis corresponds to an element of the more elaborate observatory "equatorial instrument mounting," which is designed for the telescope to follow a star's travel in diurnal circle. The solar telescope is mounted in collar bearings whose bases are attached to a vertical limb; the telescope may be revolved or turned 12 hours in hour angle.

(2) The vertical limb is an arc that is graduated to read in latitude; a vernier mounted on the base frame gives the reading in latitude; the center of the limb is called the latitude axis, and is horizontal.

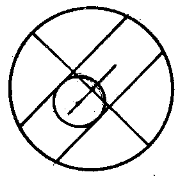
(3) A reflector at the objective end of the telescope picks up the light rays of the sun; its axis is normal to the line of collimation. An arm controls the angle of the reflector in the plane of the great circle that passes through the pole and the sun; a vernier on the arm gives the reading on a graduated declination arc.

(4) A small graduated circle on the telescope, normal to the line of collimation, reads in hour angle from 6 a.m., 7, 8, up to 12, and 1, 2, 3, up to 6 p.m.; this reads directly in *apparent time.*

(5) The plan of the reticle includes three "equatorial wires" that are set parallel to the axis of the reflector. One is in the line of collimation, the others parallel, spaced at 15′ 45″ to conform with the sun's July 1 diameter (the smallest for the year). A fourth cross wire, normal to the others, passes through the line of collimation, indicating the center of the field in *time.*



Not oriented
When turning the transit in horizontal angle, the image of the sun cuts across the equatorial wires.



Oriented
The travel of the sun's image is along the path of the equatorial wires.

Orientation of Sun's Image on Reticle

The frame of the solar unit is attached to one standard of the transit, controlled in position by three foot posts. This is the standard on the east when the transit is oriented in the meridian. The vertical plane of the polar axis may be adjusted to true parallel with the vertical plane of the transit telescope. By first setting the plate reading at zero, when preparing to orient, all horizontal angles will count from the meridian.

When oriented, and turned in hour angle to agree with the apparent time, the image of the sun will travel across the field of the solar telescope along the path of the equatorial wires. If the transit is turned away from the meridian to the right or to the left, the sun's image will cut across the equatorial wires. Thus to bring the solar unit into proper orientation, all that is needed is to see that the image of the sun is centered anywhere along the length of the equatorial wires. This centering is done with the lower tangent motion, the plates clamped at zero.

On the declination arc, an actual arc or segment of 5° is graduated for reading 10°; this is because a movement of 5° in the reflector position makes an angle of 10° between the light rays of incidence and those reflected. At zero declination the plane of the reflector is at 45° to the line of collimation. The declination arc is graduated north and south from zero for the range of the sun's position during the year.

One important element of the mounting is that the three points of control at the foot posts are placed to form a right-angle, one side of which is vertical, the other horizontal. In adjustment, the foot post at the 90° angle remains fixed. One of the foot posts controls the position of the latitude axis in horizontal. One foot post controls the direction of pointing of the line of collimation when in horizontal sighting, to bring that into parallel with the vertical plane of the transit. The foot-post controls are secured with capstan or hexagon nuts.

At one end of the frame that supports the collar bearings, there is a mechanism (corresponding to that of a telescope level) for adjusting the polar axis so that it is normal to the latitude axis. This is needed when the solar telescope is changed in latitude setting.

2–68. With the transit and solar unit in satisfactory adjustment, the simple steps in solar orientation at any setup are these: carefully level, with the solar unit on the west; reverse the instrument and correct *half* of any discrepancy in the centering of the plate bubbles; set the plates at zero; set the latitude and declination, or check the previous setting; turn the solar telescope to the reading in approximate apparent time; move the whole instrument in horizontal angle for position near the meridian, at this time bringing the sun's image into the field of the solar telescope, then tighten the lower clamp; use the lower tangent motion for final orientation, in which step the sun's image should be centered on the equatorial wires.

The solar transit is equipped for making any type of stellar or solar observation that may be employed profitably in land surveying practice as adapted to the one-minute transit. This calls for accuracy to a tolerance of $\pm 15''$ in the direction of lines, where that may be required. This accuracy is greater than that called for by the precision of distance measurement unless the character of the survey is such as to justify the greatly increased cost of exactness in measurement.

The use of the solar unit may be almost continuous as when running the line through timber or tall undergrowth, or it may be more or less incidental as when running in an open country. It is important, too, in the open country, and on almost any type of survey, to have the use of the solar unit in making the start in the line running or observing. Even on the work that requires the greatest refinement in the important lines, there are many off-line stations to be occupied for collateral data, mapping, or traversing, where the direction from true north should be employed.

For these reasons, the preparation of each day's work requires that the data shall be at hand, in the field tablet, for the sun's declination for the day, reading for value in the apparent time of the local meridian, and to which has been applied the correct refractions in polar distance for that position of the sun and that latitude. Additionally, the true latitude and the *instrumental latitude* should be known.

Proper accuracy in solar orientation becomes attainable as soon as the sun is high enough to reduce the refraction correction to not over 4′ or 5′, and continuing until 10:30 a.m. or a little later with care and suitable checks; a corresponding period applies in the afternoon. Thus from 10:30 a.m. to 1:30 p.m., or for about that period, the line running should be by back-and-foresight. If the sun becomes obscured, the usual transit methods are employed. In stopping for the day, an azimuth mark should be set for use the next morning.

2–69. *Computing Hourly Declinations of the Sun.*

For use with the solar unit, hourly declinations of the sun for each date may be prepared in tabular or graphic form, the graphic form being most advantageous.

Example of a table of hourly declinations of the sun, combined with refraction in polar distance, for March 14, 1972, at station in latitude 33°10′ N., longitude 116°45′ W. (7^h-47^m):

Declination of the sun at Greenwich apparent noon, March 14, 1972 .... 2°22′30.8″ S.

Difference in time from Greenwich apparent noon to 7 a.m., app. time, longitude 116°45′ W.

For longitude .... = 7^h47^m

For time, a.m.,

$12^h - 7^h00^m$ .... = −5 00

$2.78^h = 2^h47^m$

Hourly difference in declination .... = 59.21″

Difference in declination from Greenwich apparent noon to 7 a.m., app. time, longitude 116°45′ W.,

2.78 × 59.21 = 164.6″ .... = 2′44.6″ N.

Declination of the sun, 7 a.m., app. time = 2°19′46.2″ S.

Apparent time	True declination	Refraction	Declination setting
7 a.m.	2°19′46″ S.	2′39″ N.	2°17′07″ S.
7½	2 19 17	1 47	2 17 30
8	2 18 47	1 21	2 17 26
9	2 17 48	0 57	2 16 51
10	2 16 49	0 47	2 16 02
11 a.m.	2 15 49	0 44	2 15 05
Noon	2 14 50	0 41	2 14 09
1 p.m.	2 13 51	0 44	2 13 07
2	2 12 52	0 47	2 12 05
3	2 11 53	0 57	2 10 56
4	2 10 53	1 21	2 09 32
4½	2 10 24	1 47	2 08 37
5 p.m.	2 09 54	2 39	2 07 15

Example of a table of hourly declinations of the sun, combined with refraction in polar distance, for August 11, 1972, at a station in latitude 47°10′ N., longitude 111°00′ W. (7^h-24^m).

Declination of the sun at Greenwich apparent noon, August 11, 1972 .... 15°09′38.1″ N.

Difference in time from Greenwich apparent noon to 6 a.m., app. time, longitude 111°00 W.

For longitude .... = 7^h24^m

For time, a.m.,

$12^h - 6^h00^m$ .... = −6 00

$1.40^h = 1^h24^m$

Hourly difference in declination .... = −44.73″

Difference in declination from Greenwich apparent noon to 6 a.m., app. time, longitude 111°00′ W.,

1.40 × 44.73 = 62.6″ .... = 1′02.6″ S.

Declination of the sun, 6 a.m., apparent time .... = 15°08′35.5″ N.

Apparent time	True declination	Refraction	Declination setting
6 a.m.	15°08′35″ N.	3′28″	15°12′03″ N.
6½	15 08 13	2 21	15 10 34
7	15 07 51	1 45	15 09 36
8	15 07 06	1 08	15 08 14
9	15 06 21	0 51	15 07 12
10	15 05 37	0 41	15 06 18
11 a.m.	15 04 52	0 38	15 05 30
Noon	15 04 07	0 36	15 04 43
1 p.m.	15 03 22	0 38	15 04 00
2	15 02 38	0 41	15 03 19
3	15 01 53	0 51	15 02 44
4	15 01 08	1 08	15 02 16
5	15 00 23	1 45	15 02 08
5½	15 00 01	2 21	15 02 22
6 p.m.	14 59 39	3 28	15 03 07

Examples of diagrams showing declinations of the sun for given dates, combined with refractions in polar distance, are given in figures 10 and 11. The horizontal lines represent each hour of the day; the vertical lines represent intervals of one minute in declination. It is convenient to use the right-hand side of the sheet to represent north, the left-hand side to represent south. North declinations increase numerically to the right-hand side of the sheet, south declinations to the left-hand side. The vertical lines are numbered to suit the range of declination for the date.

The advantage of the diagram method is found in the avoidance of errors of computation and the ease with which it is checked, together with the fact that in the use of the diagram actual values are obtained at any time instead of by a linear interpretation.

Two points are marked on the diagram to agree with the true declination of the sun; the first point is marked with the argument of declination agreeing with the declination of the sun taken from the Ephemeris for Greenwich apparent noon, with the argument of time agreeing with the apparent time at the longitude of the station, corresponding to Greenwich noon; the second point is marked agreeing with the proper declination and time 10 hours later. The straight line determined by the two points agrees with the sun's true declination for the apparent time at the longitude of the station. The proper refractions in polar distance are then scaled from the straight line to the N. for each tabulated refraction, a. m. and p. m., taken from table 23, Standard Field Tables, appropriate to the latitude of the station, and declination of the sun. The latter points are then connected to form a smooth curve representing the declinations of the sun, corrected for refraction in polar distance, for use with the solar unit. The scale of the refractions must equal the scale of the intervals of 1′ in declination; the refractions are laid off along or parallel to the horizontal lines, and *not* normal to the line of true declination. At any time throughout the day the proper declination for use with the solar unit is obtained by reference to the curve at the point corresponding to the time of observation. To obtain any true value of the sun's declination for use in the reduction of *altitude observations* reference may be made to the

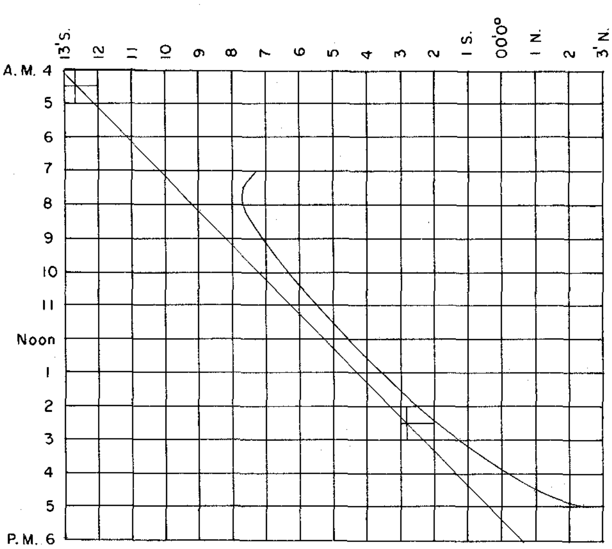


FIGURE 10.—Diagram of sun's declinations.
Date, March 20, 1970.
Station: Latitude 37°30′ N. Longitude 112°30′ W. (7^h30^m)
Declination at Greenwich app. noon
(4^h30^m a.m., app. time) = 0°12′39.3″ S.
Difference in declination for 10^h = 592.5″ = 9′52.5″ N.
Declination at 2^h30^m p.m., app. time = 0°02′46.8″ S.

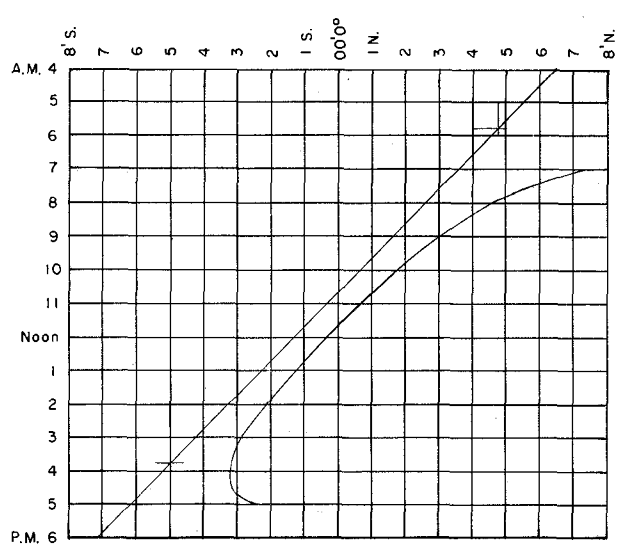


FIGURE 11.—Diagram of sun's declinations.
Date, September 23, 1971.
Station: Latitude 47°30′ N. Longitude 94°30′ W. (6^h18^m)
Declination at Greenwich app. noon
(5^h42^m a.m., app. time) = 0°04′44.0″ N.
Difference in declination for 10^h = 584.1″ = 9′44.1″ S.
Declination at 3^h42^m p.m., app. time = 0°05′00.1″ S.

straight line of true declination at the point corresponding to the time of observation.

Apparent Time from the Solar Unit

2–70. The solar unit of the solar transit has a graduated hour circle mounted normal to the polar axis. The readings are indicated at intervals of 10 minutes from 6 a.m. to 6 p.m., apparent time; the readings may be estimated to about ±1 or 2 minutes. This accuracy is sufficient for taking out the sun's declinations, which are calculated in terms of apparent time. It can be an approximate check upon the altitude and meridian observations on the sun for apparent time.

By applying the equation of time to the reading of the hour circle, the watch may be set to approximate local mean time, with the tolerance indicated above. This will be accurate enough for the finding positions for the stellar observations. It is also sufficiently accurate for the observations on Polaris for azimuth at elongation or latitude at culmination.

An exact method for time determination should be followed for the watch correction in local mean time necessary for hour angle observations on Polaris.

Checks of the Solar Unit

2–71. The following checks of the solar unit are all that are ordinarily required at the beginning of a survey:

(1) The reading of the latitudinal vernier with the solar telescope in true horizontal position.

(2) The reading of the latitude arc at noon with the solar telescope oriented on the meridian and the correct declination setting. This is the "instrumental latitude."

(3) The reading of the declination vernier when set in true zero declination, 15° north declination, and 15° south declination.

(4) The check for parallelism when the solar telescope is set and clamped in the latitude of the station.

(5) The checks for orientation when compared with a true meridian.

The solar unit should be checked on a true meridian at least weekly in normal use and whenever it has been subject to unusually hard bumps or jars.

2–72. Reference should be made to the maker's bulletin or a surveying textbook for care and adjustment of the transit. The adjustments of the solar unit are described in appendix I.

Errors in Azimuth, Due to Small Errors in Declination or Latitude

2–73. It may frequently happen with a solar transit, especially at the beginning of a survey or with an instrument insufficiently tested, that the first meridional trials are made with slight errors in the settings of the latitude and declination arcs, resulting in small errors in azimuth. This may be particularly true prior to a determination of the instrumental latitude. The discrepancies in azimuth due to such errors have been tabulated in the Standard Field Tables, which may be applied to results of single observations with considerable certainty. The corrections are not applicable to a series of observations as in ordinary line work owing to the changing values (for hours from noon) of the correction coefficients. The explanation with the table gives a key to the direction of the azimuth errors on account of small incorrect values in setting the latitude and declination arcs.

For example, at 9^h40^m a.m., app. t., at a station in latitude assumed to be 46° 20′ N., a test was made with a solar transit whereby the trial indication was found to be S. 0° 05′ W., or 0° 05′ west of the true meridian. Subsequent determinations of the true latitude of the station and of the correctness of the vernier of the declination arc showed that the actual latitude of the station was 46° 21′.5 N., and that the vernier of the declination arc had an index error which gave readings 0° 00′.5 S. of the calculated declination (i.e. reading 15° 19′.5 N. for a calculated declination of 15° 20′ N.) Thus in the test the latitude arc was set 1′.5 S. of the correct latitude of the station, and the declination arc was actually set 0′.5 N. of the value that would have been set had the index error been known.

By reference to the Standard Field Tables:

Latitude	Hours from noon			
	2^h0^m	2^h20^m	3^h0^m	
45° 00′	2.83	2.55	2.00	
46 21.5		2.62		Declination coefficient.
50 00	3.11	2.81	2.20	
45 00	2.45	2.10	1.41	
46 21.5		2.16		Latitude coefficient.
50 00	2.69	2.31	1.56	

The corrections are then applied as follows:

Indication of solar in test = S. 0° 05′.0 W.
Correction for declination = 0 01 .3 E. = (2.62 × 0.5)
Correction for latitude = 0 03 .2 E. = (2.16 × 1.5)

Corrected indication of solar = S. 0° 00′.5 W.

The above corrections will often serve to explain the apparent errors of the solar transit, but are not intended for use in line work, and cannot be accepted in lieu of subsequent tests based on correct values.

THE GEODESY OF LARGE-SCALE CADASTRAL SURVEYS

Transfer of Azimuth, Station Error, and Curvature

2–74. When carrying forward the direction of lines through intermediate transit stations by the method of fore-and-back sights and deflection angles, two corrections become important where the purpose is to maintain accuracy. First, each station setup involves uncertainty in the maintenance of the direction of a line, or in the value of the angle that may be turned, called "station error." Second, if the line is other than a meridian, its direction will have an increment of *curvature;* this is applied in order to convert from the forward azimuth to the back azimuth of that same line at the next station.

As solar transit orientation is designed to give the meridian at each station, thereby avoiding cumulative errors of conventional transit methods, the corrections for station error and curvature do not enter into the ordinary solar transit directions. However, for the purpose of a comparison of the solar transit direction *of the chord* of a long line, *half* the value of the convergency of the meridians of the two end stations is applied.

For example, a parallel of latitude as run by solar transit methods is a true latitudinal curve, i. e.—a small circle of the earth, everywhere due east or west. The transit line or chord between any two distant points of the parallel is a great circle, whose mean azimuth, or bearing at midpoint, is due east and west. At one end of the chord the forward azimuth is always northeasterly (or northwesterly); at the opposite end, the back azimuth will be northwesterly (or northeasterly). At the end stations of the chord, the difference between the forward (or back) azimuth *and due east or west,* will be equal to half the value of the curvature counting from the two end stations. At the end stations of the chord, the difference between the forward azimuth and the back azimuth ±180° will be the full value of the convergency of the meridians of the two end stations.

By basic law, and the Manual requirements, the directions of all lines are stated in terms of angular measure referred to the true north (or south) at the point of record. Therefore, after carrying a transit line forward a considerable distance through a number of intermediate stations, it is necessary to solve the problem of the change from the starting direction to that at the last station. The back azimuth at the last station ±180° should equal the starting direction, plus or minus the algebraic sum of the deflection angles, plus or minus the value of the convergency of the meridians from the two end stations. The discrepancy will be the accumulated error in the transit operations.

To determine this discrepancy, the first step is to make dependable azimuth observations at the two end stations; then compare the values, allowing for the convergency of the meridians of the end stations. The difference that remains is the *cumulative* error. The latter value, in terms of seconds, may be divided by the number of the intermediate stations; the quotient is the cumulative error per station, or just "station error." The latter should be distributed according to the number of the intermediate stations.

The total curvature, or correction for convergency is an element of the *departure* between the end stations, and of the mean latitude. It is tabulated in the Standard Field Tables, for the value in angular measure for *a departure of six miles,* or 480 chains, for each degree of latitude. It is often convenient to convert the curvature to a value for a departure of 100 chains, for proportional reduction to other distances. The whole convergency should be distributed in proportion to the *departure of each course.*

A transit line running easterly will curve to the right in bearing angle, thus increasing a northeasterly bearing, or decreasing a southeasterly bearing. A transit line running westerly will curve to the left in bearing angle, thus increasing a northwesterly bearing, or decreasing a southwesterly bearing.

When computing latitudes and departures, and transferring a geographic position by means of a long connecting line, *the mean azimuth* should be employed for the direction of the line, i.e.—the mean between the forward azimuth and the back azimuth ±180°. That azimuth or bearing angle will be the direction of the chord of the great circle that passes through the ends of the connecting line.

Where the transfer of azimuth is by triangulation, a check is secured by the closure of each triangle, and the reductions of the lengths of the lines.

The angles employed for the calculations of the lengths of lines will be the differences in the directions between the two forward azimuths at each station.

The sum of the three angles should close to 180° within the allowable tolerance.

The *mean course* of any side (actually the mean bearing of the chord) will be the mean between the forward and the back azimuth ±180° of that line.

The correction for curvature of the longest line in easting or westing in any triangle should equal the sum of the corrections for curvature of the other two sides.

As a check on the lengths of lines, the latitude of the longest line in any triangle should equal the sum of the latitudes of the other two sides.

The distances in departure must be reduced first to the mean position in latitude of each side.

A check is secured by reducing each departure to a common position in latitude. In this reduction, the amount to be added to or subtracted from each departure is equal to the amount of the distance along the meridian between the two latitudinal lines at the transfer, multiplied by the tangent of the angle of convergency.

After reducing to a common latitude, the departure of the longest line in easting or westing in any triangle should equal the sum of the departures of the other two.

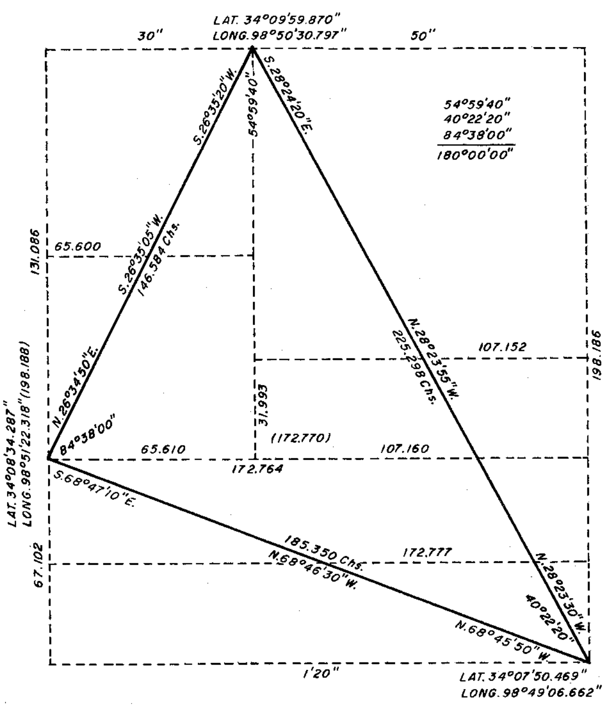


FIGURE 12.—Curvature of lines of a large triangle.

The one-minute transit, carefully handled, is capable of holding the station error to under 10″ per station, or even to about half that amount if the nature of the survey requires a very high degree of accuracy, and there is justification for the consequent needed increase in care.

Where the survey is made with the one-minute transit, it is inappropriate to represent the computed result of an observation closer than 30″, 15″, or 10″, according to the strength of the observation.

A tolerance of 10″ in the direction of a line calls for a measurement that is good to within 1:20,000; 15″, to within 1:13,333; 30″, to within 1:6,667; 1′ 00″, to within 1:3,333; 1′ 30″ (the Manual tolerance for solar transit orientation) to within 1:2,300. This comparison will emphasize the point that in land-surveying practice, and particularly in the subdivision of large areas as in the rectangular survey of the public lands, more stress should be placed on accuracy of distance measurement if those values are to be as good as the values required in the direction of lines.

The True Parallel of Latitude

2–75. The base lines and standard parallels of the rectangular system are established on the true parallel of latitude; the random latitudinal township boundary lines are also projected on the same curve; this curve is defined by a plane at right angles to the earth's polar axis cutting the earth's surface on a small circle. At every point on the true parallel the curve bears due east and west, the direction of the line being at right angles to the meridian at every point along the line. Two points at a distance of 20 chains apart on the same parallel of latitude may be said to define the direction of the curve at either point, without appreciable error, but the projection of a line so defined in either direction, easterly or westerly, would describe a great circle of the earth gradually departing southerly from the true parallel. The great circle tangent to the parallel at any origin or reference point along the parallel is known as the "tangent to the parallel," and it is coincident with the true latitude curve only at the point of origin. The rate of the change of the azimuth of the tangent is a function of the latitude on the earth's surface. The azimuth of the tangent varies directly as the distance from the origin, and the offset distance from the tangent to the parallel varies as the square of the distance from the point of tangency. A great circle connecting two distant points on the same latitude curve has the same angle with the meridian at both points and the azimuth of such a line at the two points of intersection is a function of one-half the distance between the points.

There are three general methods of establishing a true parallel of latitude which may be employed independently to arrive at the same result: (1) the solar method, (2) the tangent method, and, (3) the secant method.

Solar Method

2–76. The solar instruments are capable of following the true parallel of latitude without substantial offsets. If such an instrument, in good adjustment, is employed, the true meridian may be determined by observation with the solar unit at each transit point. A turn of 90° in either direction then defines the true parallel, and if sights are taken not longer than 20 to 40 chains distant, the line so established does not appreciably differ from the theoretical parallel of latitude. The locus of the resulting line is a succession of points each one at right angles to the true meridian at the previous station. There are periods each day, however, when solar observations are not practicable or the sun may be obscured. Moreover, the instrument available might not have a solar attachment or one in proper adjustment. In these circumstances reference must be made to a transit line from which to establish the true latitude curve by one of the alternative methods given below.

Tangent Method

2–77. The tangent method for determination of the true latitude curve consists in establishing the true meridian at the point of beginning, from which a horizontal deflection angle of 90° is turned to the east or west, as may be required; the projection of the line thus determined is called the tangent. The tangent is projected six miles in a straight line, and as the measurements are completed for each corner point, proper offsets are measured north from the tangent to the parallel, upon which line the corners are established.

Azimuths of the tangent to the parallel, referred to the true south, are given in the Standard Field Tables. They are tabulated for any degree of latitude from 25° to 70° N., for the end of each mile from 1 to 6 miles. At the point of beginning the tangent bears east or west, but as the projection of the tangent is continued the deviation to the south increases in accordance with rules already stated.

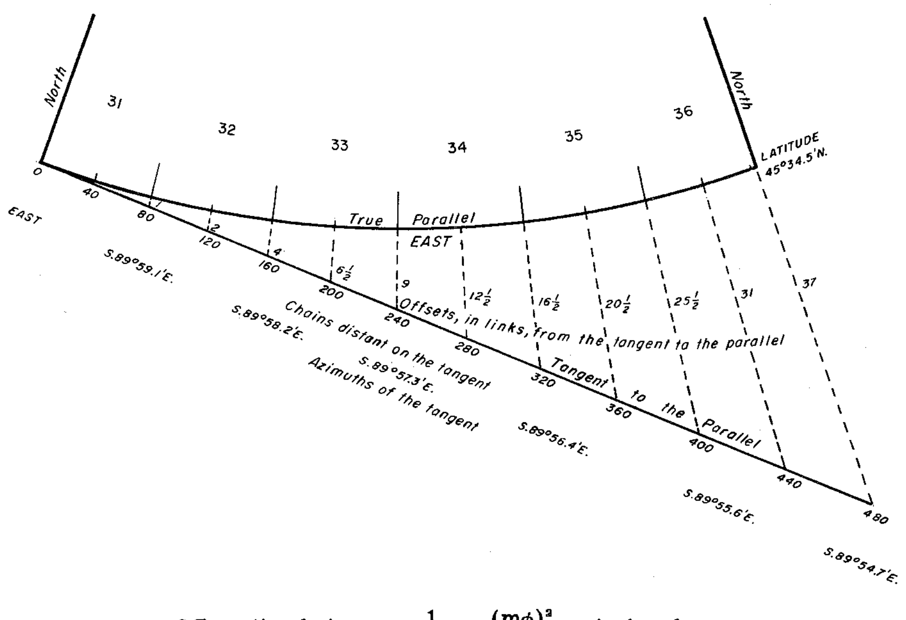
The Standard Field Tables show the various offsets from the tangent north to the parallel, tabulated for any degree of latitude from 25° to 70° N., for each half mile from ½ to 6 miles.

Figure 13 illustrates the establishment of a standard parallel in latitude 45°34.5′ N., by the tangent method.

Objection to the use of the tangent method in a timbered country is found owing to the requirement that all blazing is to be made on the true surveyed lines. Also, all measurements to items of topography entered in the field notes are to be referred to the true established lines. These objections to the tangent method, on account of the increasing distance from the tangent to the parallel, are largely removed in the secant method.

Secant Method

2–78. The designated secant is a great circle which cuts any true parallel of latitude at the first and fifth mile corners, and is tangent to an imaginary latitude curve at the third mile point. From the point of beginning to the third mile corner the secant has a northeasterly or northwesterly bearing; at the third mile corner the secant bears east or west; and from the third to the sixth mile corners the secant has a southeasterly or southwesterly bearing, respectively, depending upon the direction of pro-



$$\text{Offset (in chains)} = \frac{1}{R_p} \cdot \frac{(m\phi)^2}{2} \cdot \sin b, \text{ where}$$

$\frac{1}{R_p}$ is taken from the table in section 2–79 for the latitude of the beginning point

$m\phi$ = distance from the beginning point in chains

b = forward bearing at the beginning point

FIGURE 13.—A tangent to the parallel.

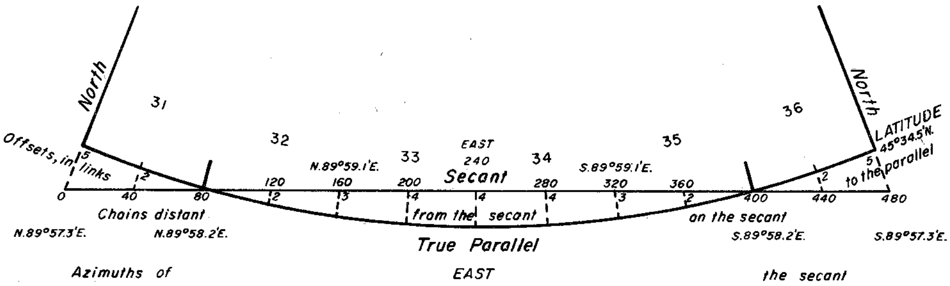


FIGURE 14.—A secant of the parallel.

jection, east or west. From the point of beginning to the first mile corner and from the fifth to the sixth mile corners the secant lies south of the true parallel, and from the first to the fifth mile corners the secant lies north of the true parallel. It will thus be seen that the secant method is a mere modification of the tangent method, so arranged that the minimum offsets can be made from the projected transit line to the established true parallel of latitude.

The secant method of determination of the true latitude curve consists in establishing the true meridian at a point south of the beginning corner a measured distance taken from the table, from which meridian the proper horizontal deflection angle, as taken from the table, is turned to the northeast or northwest to define the secant. The secant is projected 6 miles in a straight line, and as the measurements are completed for each corner point, proper offsets are measured, north or south, from the secant to the parallel, upon which parallel the corners are established.

The Standard Field Tables give the bearing angles or azimuths of the secant, referred to the true N. point for the first 3 miles, and the same symmetrical bearing angles or azimuths referred to the true S. point for the last 3 miles, tabulated for any degree of latitude from 25° to 70° N., for the end of each mile from 0 to 6 miles.

The Standard Field Tables show the various offsets from the secant to the parallel, tabulated for any degree of latitude from 25° to 70° N., for each half mile from 0 to 6 miles.

Figure 14 illustrates the establishment of a standard parallel in latitude 45°34.5′ N. by the secant method.

The secant method is recommended for its simplicity of execution and proximity to the true latitude curve, as all measurements and cutting by this method are substantially on the true parallel.

Convergency of Meridians

2–79. The linear amount of the convergency of two meridians is a function of their distance apart, of the length of the meridian between two reference parallels, of the latitude, and of the spheroidal form of the earth's surface.

The following equation is convenient for the analytical computation of the linear amount of the convergency on the parallel, of two meridians any distance apart, and any length. The correction for convergency in any closed figure is proportional to the area, and may be computed from an equivalent rectangular area:

"m_λ": Measurement along the parallel.
"m_Φ": Measurement along the meridian.
"a": Equatorial radius of the earth=3963.3 miles.
"e": Factor of eccentricity, log e=8.915 2515.
"dm_λ": Linear amount of the convergency on the parallel, of two meridians distance apart "m_λ" and length "m_Φ" along the meridian: "dm_λ" "m_λ" "m_Φ" and "a" to be expressed in the same linear unit:

$$dm_\lambda = \frac{m_\lambda m_\Phi}{a} \tan\phi \sqrt{1 - e^2 \sin^2\phi}$$

Example of computation of the convergency of two meridians 24 miles long and 24 miles apart in a mean latitude of 43°20′:

nat 1	=		1.0000000
log e	=	8.915 2515	
" "	=	8.915 2515	
" sin 43° 20′	=	9.836 477	
" " " "	=	9.836 477	
" $e^2 \sin^2 \phi$	=	7.503 457	
nat " " "	=		0.0031875
" $(1 - e^2 \sin^2 \phi)$	=		0.9968125
log " " "	=	9.998 614	
" $\sqrt{1 - e^2 \sin^2 \phi}$	=	9.999 307	
" tan 43° 20′	=	9.974 720	
" 24	=	1.380 211	
" "	=	1.380 211	
" 80 [2]	=	1.903 090	
" product	=	4.637 539	
" 3963.3	=	3.598 057	
" dm_λ	=	1.039 482	
nat "		10.9517	chs.

The above equation may be written $dm_\lambda = \frac{1}{R_p} m_\lambda\, m_\Phi$, where $R_p = \frac{a \cot \phi}{\sqrt{1 - e^2 \sin^2 \phi}}$. The computed linear convergence will be in the same unit as m_λ and m_Φ. Values of $\frac{1}{R_p}$ for latitudes 25° to 75° are listed in the table below.

Lat.	$1/R_p$	Lat.	$1/R_p$	Lat.	$1/R_p$
25°	.000001470	42°	.000002836	59°	.000005236
26	.000001537	43	.000002937	60	.000005449
27	.000001606	44	.000003041	61	.000005675
28	.000001676	45	.000003149	62	.000005916
29	.000001747	46	.000003260	63	.000006173
30	.000001819	47	.000003376	64	.000006449
31	.000001893	48	.000003496	65	.000006745
32	.000001969	49	.000003621	66	.000007064
33	.000002046	50	.000003751	67	.000007409
34	.000002125	51	.000003887	68	.000007784
35	.000002206	52	.000004028	69	.000008192
36	.000002289	53	.000004176	70	.000008640
37	.000002374	54	.000004331	71	.000009132
38	.000002461	55	.000004494	72	.000009677
39	.000002551	56	.000004665	73	.000010284
40	.000002643	57	.000004845	74	.000010965
41	.000002738	58	.000005035	75	.000011734

Using the same values as in the previous example:

$dm = .000002971 \times 1920 \times 1920 = 10.952$ chains.

The convergency, measured on the parallel, of two meridians 24 miles apart and 24 miles long, in a mean latitude of 43°20′, is therefore found to 10.95 chains. The convergency of the east and west boundaries of a regular township in the same latitude would be equal to one-sixteenth of the convergency of the east and west boundaries of the quadrangle as computed above, or 68.44 links, which agrees with the value taken from the Standard Field Tables.

2–80. The Standard Field Tables list the linear amounts of the convergency of meridians, six miles long and six miles apart, for each degree of latitude from 25° to 70° N., together with the angle of convergency of the same meridians. These amounts of linear convergency are at once the proper corrections to apply to the north boundary of a regular township in the computation of the closing error around a township, or other computation by which a theoretical length of a north or south boundary of a township is compared with the length of the opposite boundary; the tabulated linear amounts of convergency are equal to double the amounts of the offsets from a tangent to the parallel at six miles for the same latitudes. Simple interpolation may be made for any intermediate latitude, and the amount of the convergency for a fractional township or other figure may be taken in proportion to the tabulated convergency as the fractional area is to 36 square miles.

The tabulated angle of convergency represents at once the deviation in azimuth of the tangent from the parallel at six miles; and $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$ of the tabulated angles of convergency represent at once the amounts of the correction in the bearing of meridional section lines to compensate for convergency within a township.

In the same table are given the differences of longitude for six miles in both angular and time measure, also the differences of latitude for one or six miles, in angular measure, in the various tabulated latitudes.

In the plan of subdivision of townships the meridional section lines are established parallel to the east boundary or other governing line. This necessitates a slight correction on account of the angular convergency of meridians. Meridional section lines west of the governing line are deflected to the left of the bearing of the governing line the amount shown in

[2]This factor is introduced here for the purpose of conversion from the unit expressed in miles to the unit expressed in chains.

the Standard Field Tables, which is entered under two arguments: (1) latitude, and (2) distance from the governing line. Meridional section lines east of a governing boundary are given the same amount of correction for bearing but the deflection is made to the right.

Lengths of Arcs of the Earth's Surface

2–81. All computations involving a difference of latitude for a given measurement along a meridian or the converse calculation, or other computations involving a difference of longitude for a given measurement along a parallel or a similar converse calculation, are readily accomplished by use of the values given in the Standard Field Tables. The table gives the lengths in miles and decimal part of a mile of one degree of longitude measured on the parallel, and the lengths in miles of one degree of latitude measured on the meridian, for any latitude from 25° to 70° N.

The tabulated values may be reduced to miles and chains, or to chains or feet, as convenient. In taking out lengths of degrees of longitude measured on the parallel the value should be taken out for the mean position in latitude of that portion of the meridian whose length is to be computed.

Geographic Positions

2–82. The term "geographic position" is used interchangeably with "geodetic position," and refers to a position on the spheroid representing the earth. The spheroid in general use in North America is Clarke's Spheroid of 1866. It is defined by the dimension of its equatorial axis and the ratio to this length of the amount it exceeds the polar axis. The spheroid closely approximates the shape the earth would have if the ocean surface were continuous. Any position is defined by its latitude and longitude measured from the intersection of a meridian through Greenwich and the equator. It will be seen that linear measurements made between geodetic positions must be reduced to sea level to check the theoretical distance. In the ordinary cadastral survey this refinement will generally not be necessary.

In the township plats of the rectangular system it is the practice to give the geographic position of the southeast corner as determined from the best available source. The surveyor should tie his work to horizontal control stations of the National Geodetic Survey and the United States Geological Survey whenever practicable.

Geodetic control has assumed increased importance to the cadastral surveyor with the use of protractions to define parcels of unsurveyed land.

Connections from land lines to geodetic control may be made by intersection from transit stations while the survey is in progress or by a transit traverse to the control station.

The Standard Field Tables include a tabulation of "M and P factors" for use in converting distances in chains to differences in latitude and longitude. "M" refers to distances along the meridian and "P" along the parallel. Distances in chains multiplied by the M or P factor for the appropriate latitude give the differences in latitude or longitude in seconds.

Differences in latitude are computed first. If the distance is large, the latitude of the unknown position is estimated, and the M factor is taken from the table for the mean latitude. In using the P factor, an interpolation to seconds of latitude is made for each segment of the line, as this factor changes rapidly.

Plane Coordinates

2–83. The local surveyor is often concerned with feet, rather than chains. Townsite surveys, highway surveys, and surveys for other engineering projects are made in feet. It is convenient for purposes of a permanent record of positions on such work, and as a check on the closures obtained, to make use of the State coordinate systems.

The State coordinate systems are rectangular grids designed to fit the curved shape of the earth to a plane surface with as little distortion as possible. By choosing a limited area and a conformal projection, this is accomplished. The State systems are based on either the Transverse Mercator or the Lambert projection. Use of such a system depends on the availability of a sufficient number of geodetic control monuments to permit the determination of the grid position of points in the survey by plane surveying.

The scale error varies from zero up to about one part in 10,000. The grid azimuth is a true azimuth only along the central meridian of the zone. Any point can be reestablished once its coordinates have been determined.

The State coordinate systems are used in the definition of some mineral leasing blocks on the outer continental shelf. Other applications are in photogrammetric and electronic surveys.