5/14/93

A CAD APPROACH TO MEDIAN LINE COMPUTATIONS

Jerry L. Wahl
and
Tom Noble
Bureau of Land Management
Cadastral Survey

ABSTRACT

The problem of computing a riparian median line is one that most surveyors rarely encounter. It is one of the more labor intensive analytical computation processes in surveying. Evolving technology in computations and Computer Aided Design (CAD) systems have placed in the hands of the surveyor tools that allow us to easily and accurately visualize the problem, verify solutions, and provide an efficient means to obtain solutions. This paper discusses the basics of median line computations and describes some of the AutoCad tools that have been developed to simplify accurate median line computations.

GENERAL

A median line is technically defined as a line midway between two shores or banks. It can be defined by key points known as *salient points* along the shore. Another way is to use record meanders defining the approximate banks. The problem of computing a line between two banks might seem like a simple problem to solve, but closer examination of the problem indicates that in actual practice the median can be a very complex *line* that involves many straight and parabolically curved segments. This paper involves a technical exploration of median line computation.

Median lines can be determined within a variety of bodies of water. Some international and state boundaries in oceans, seas as well as rivers are determined by median lines. Lakes as well as rivers may have boundaries defined in them by median lines. Boundaries within relicted bodies of water are also frequently defined by median lines.

Median line computation has been discussed in a number of papers, primarily in the application to mapping international boundaries and frequently from a cartographic rather than survey viewpoint. (see references)

PHILOSOPHICAL CONSIDERATIONS

Arbitrariness: Water boundary issues frequently possess an element of arbitrariness. By this we mean that the actual truth cannot be precisely known or measured, or the problem is too complex to deal with in absolute terms. Any computational process involving riparian issues must straddle this factor. Median line computations are no exception to this philosophical dilemma in that a very detailed, complex and precise computation proceed from rough and sometimes crude base data. For example, we can define the margin of a body of water by salient points scaled from a 1:100,000 chart, or we can use the record of an old survey of a meander which miscloses by hundreds of feet. On the other extreme we can perform a new meander, or we can use large scale photography to deduce all the nooks and crannies along the shore to the 'grain of sand' level. You also may be dealing with an historical shore line that no longer exists and therefore cannot be measured more accurately and whose location may in fact be a matter of opinion and is frequently disputed. Nevertheless a precise computation has to proceed from the imprecise defining data.

Certainty: After the base data is agreed upon or stipulated, there is only one true possible median line solution for that system. The median 'line' may be very complex combination of straight line segments and parabolic curves, some only a few feet in length. It may be extremely time consuming to determine and compute so that in the end it is not often popular to perform a complete solution.

CONTROLLING ELEMENTS

A median line can be determined in several ways. One of the more common methods is by means of salient points.

Salient Points. A salient point is a prominent point along the shore of the water body that has a dominant affect on the median line. The median line is then determined by selection of pairs of points and computing the perpendicular bisector between them.

Meander Lines. Another way that a median line can be determined is by using meanders. Meanders are traverses, courses and distances, surveyed to roughly define the banks of bodies of water. In public land surveying within the Bureau of Land Management we are guided by the *The Manual of Surveying Instructions, 1973*, which in Section 7-55 states:

> *"The simplest method of determining the median line is by use of salient points. This method is of limited application in public land surveys, where the median line is nearly always determined from the meander lines."*

Thus the general solution we are faced with in PLSS cases is a meander determined median line. In the course of investigating how to compute a meander determined median line, it becomes clear that the salient point solution method is a subset of the meander solution methods.

As in the case of salient points, the data you use for the position of the meanders may seem arbitrary, but once settled the median line is set.

ELEMENTS OF A MEANDER DETERMINED MEDIAN LINE

Basic Elements of Median Lines

Any given point on the median line is controlled by one element on each of the meanders. These controlling elements are either lines or points. The points may be salient points or the angle points of meander line. Therefore the possible controlling conditions are:

Line opposite a line. Referred to as a the **Line-Line** condition. The median line is a straight line midway between the two banks whose position and direction is defined by determining the angular bisector of the two bank line segments.

The condition ends when it intersects another condition or when one of the controlling lines ends. That point is at the intersection the first perpendicular from the ends of the controlling lines.

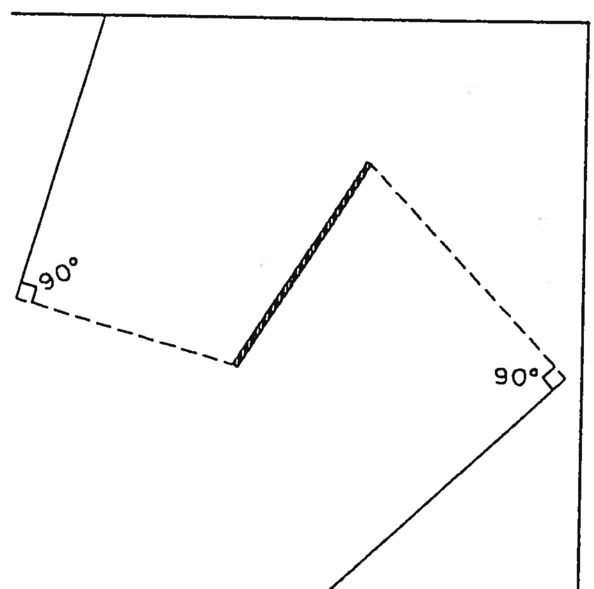

Figure 1: Line-Line

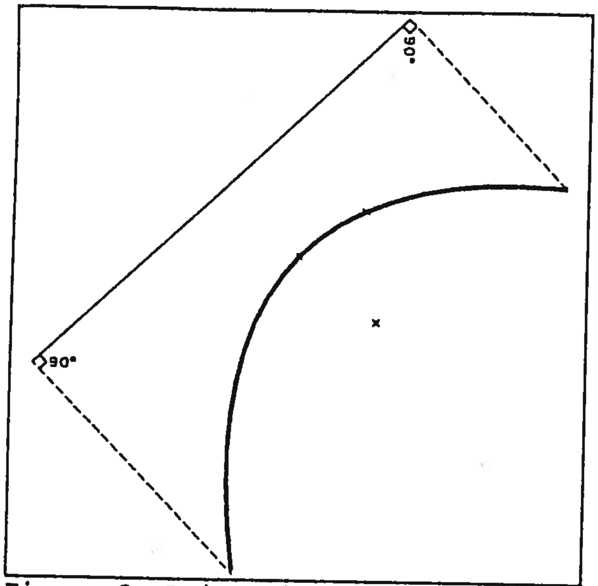

Figure 2: Point-Line

Line opposite a point. This is the **Line-Point** condition. The median line is a parabolic curve whose directrix is the controlling line, and whose focus is the opposite controlling point. A parabola is defined as the locus of points midway between a point (the focus) and a line (the directrix). Although the Manual seems to almost say that these are simple curves, they are not.

The condition ends when the median line intersects another segment or when the controlling line ends. That end point on the median line is at points perpendicular from the controlling line's endpoint.

Point opposite a point. Referred to as a **Point-Point** condition. This is identical to a salient point computation. The solution for the point-point condition is a straight line segment that is a portion of the perpendicular bisector to a line between the two points. In the case of a meander determined median line, the condition ends at the first perpendicular it runs into from one of the lines into the points.

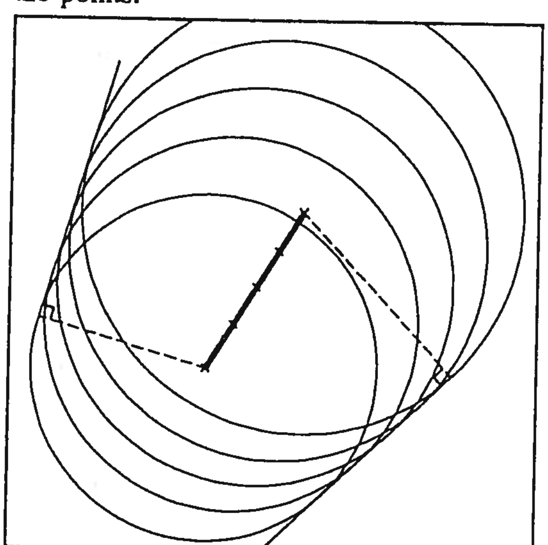

Figure 4: L-L by Circles

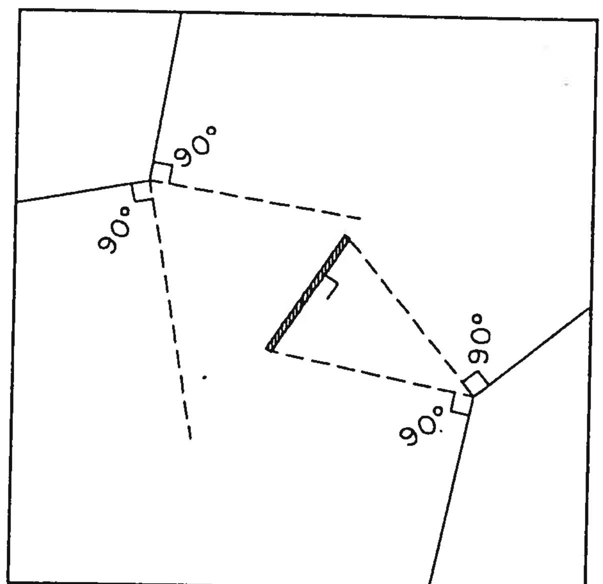

Figure 3: Point-Point

A visual analogy that can be useful in thinking about a median line is to think of the median line as the line swept out by the center of a circle of varying radius travelling down the channel between the two banks. This leads to the following descriptions of the same conditions.

The **line-line** condition is visualized as the circle is sliding between the two lines, squeezing or expanding in size. The condition terminates when

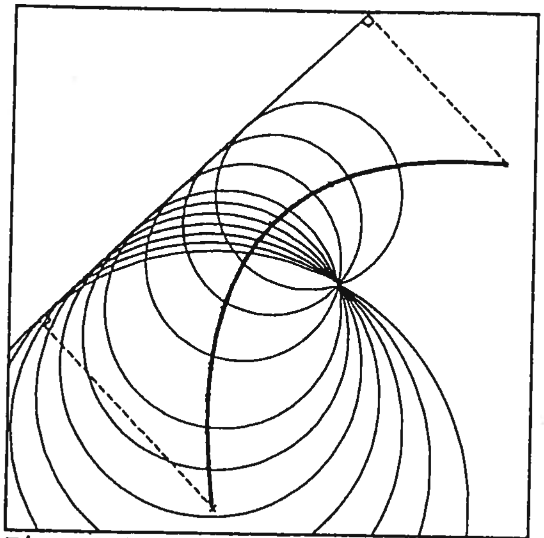

Figure 5: P-L by Circles

the circle bumps into another line or point along the bank or else rolls off one of the lines onto an angle point.

The point line condition is visualized as a circle pivoting on the controlling point as it squeezes by the point tangent to the opposite bank. The condition terminates when the circle bumps into another portion of one of the banks, or when it rolls off of the point onto one of the lines. That point will be at the first perpendicular from a line into the point.

The point-point condition is visualized as a circle squeezing down between the two points. The condition terminates when the circle rolls off of the point and onto one of the lines into the point at the first perpendicular from those lines.

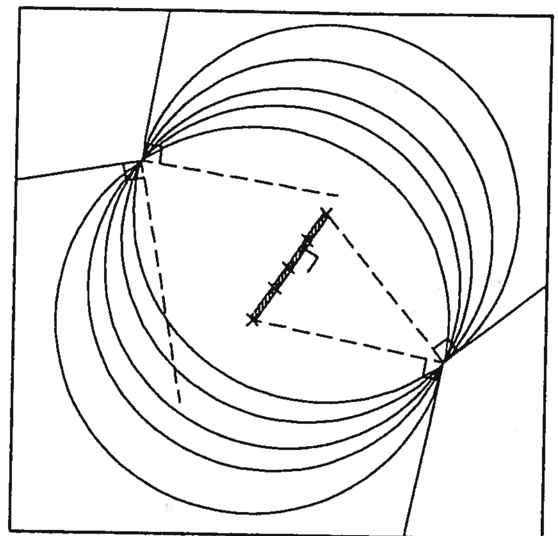

Figure 6: P-P by Circles

__Complete Solution__. The solution to a median line is simply the application of the above basics and comes down to determining the proper controlling condition at any given point along the median line. This seems fairly easy in the examples shown above, but in actual practice it can be very difficult.

Determining what the controlling elements are can be deceptive to say the least. This difficulty becomes more acute the more detailed the meanders are, or the further apart the banks are in relation to the average meander length. In addition, the solution is complicated by all the combinations of intersecting conditions. The combinations are:

1: L-L Line intersects L-L Line
2: L-L Line intersects L-P Parabola
3: L-L Line intersects P-P Line
4: L-P Parabola intersects L-P Parabola

The above condition can be the parabolic equivalents to a compound curve or a reverse curve.

5: L-P Parabola intersects P-P Line
6: P-P Line intersects P-P Line

The traditional process for developing a median line usually starts with a sketch drawn to scale. This is done at as large a size as possible to allow a graphic determination of what appear to be the controlling conditions. Then arcs are thrown with a compass and candidates for possible controlling conditions are developed. Once that is done, a mathematical solution can be computed for those candidates. It becomes a process of trial and error. Sometimes it is difficult to tell exactly what is happening and it is best to simplify the problem by selecting major points along the meanders rather than attempting to solve the median in minute detail.

Once points are computed they can be plotted back on the sketch and tested to see if they appear to fall on the median line. More often than not the test shows that the assumed condition is not correct and additional options have to be tried until a reasonable looking result is obtained. It is a trial and error process which can be very time consuming, and a lot of skill and experience becomes an asset.

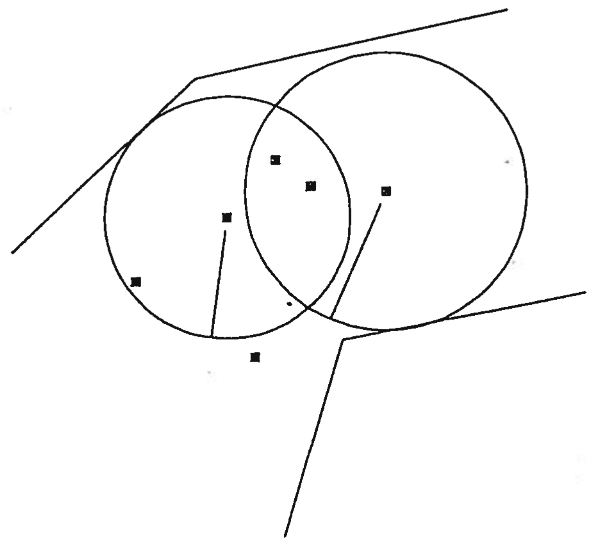

Figure 7 - Checking Bad Points

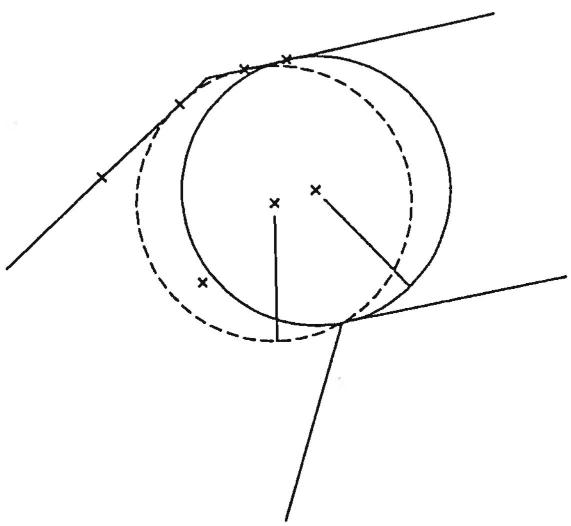

Figure 8 - Checking Good Points

In 1988 the BLM California Cadastral Survey began to explore the use of computer graphics in the form of a CAD system as an aid in median line computation. At a minimum the system would provide an accurate sketch and eliminate many of the problems associated with hand drawn sketches such as drafting, scaling, the imprecision in graphic estimating, the problem of multiple erasures coming from the many trials needed to determine the final set of conditions.

CAD as an evaluation tool:

It was quickly apparent that AutoCad, the CAD system we were using, would be a big aid to the process. The second advantage of the CAD system approach was the ability to test trial solutions. Having a very accurate diagram provided an easy means of testing median point candidates by throwing circles from them. This simple methodology provided an absolute check if a point was a valid point on the median line. Examples are shown in Figure 7 and Figure 8.

If the circle intersected the controlling meanders segments exactly, and no others, then it was proved to be a valid point on the median line. Because of the zoom capabilities of CAD packages it is easy to determine if the intersections are close to a degree not possible with a hand drawn diagram.

CAD AS A COMPUTATION TOOL

Custom AutoCad Median Line Functions. The next step in the use of the CAD system was to begin actually performing some of the condition computations. AutoCad provides an internal user programming language AutoLISP. We had made extensive use of this capability in our plat drafting system. It seemed a logical approach to develop a few functions to solve at least the more obvious median line situations. It should also be noted that AutoCad provides very accurate computation capabilities, and that the current BLM Cadastral plat drafting system are functions for bearing-distance line entry, line labelling, grant and compass adjustments, area computations and many other functions. A few functions were then programmed to perform quick computations for the 3 conditions described above. They were:

PULL function. For the Line-Line condition, this function allowed quick construction of many of the straight line segments. Intersections with other line conditions were obvious and could be derived directly with AutoCad.

PARA function. For the Point-Line condition, this function was developed as an aid to construction of the curved segments. Since these segments are actually parabolic curves they pose a greater problem. First, AutoCad has no direct means to draw a parabola, however a function called *PARA* was created that computes points along a parabola at intervals defined by the user and connects them with short straight segments, in essence small chords. In addition AutoCad provides curve and spline fitting methods which can be used to very closely approximate the parabola between the computed points. The user can select the interval to use: long to speedup the process, or short provide more detail as desired.

This function made it much easier to visualize what was happening with the curve conditions. For example curve intersecting curve conditions could be easily visualized, (see ***Figure***). It was also apparent that an exact solution for the curve intersecting anything condition was not always possible even with this enhanced graphic approach since the intersection was most likely to be somewhere along the small chord rather than on an exact parabola. This error could become unacceptable when the intersection was with another curve or line segment at very nearly the same bearing.

MPP Function. Is the Median Point-Point condition function. It draws a portion of the median line that can be trimmed or extended to it's proper terminus.

MCHK Function. Median Check. This was developed as a simple routine that automated the creating of a check circle by incorporating a few native AutoCad commands.

DROP Function. Drops a perpendicular from a point to the selected meander line, as an aid in determining the beginning point for new conditions after an intersection has been found.

Intersecting Conditions. The exact solution for most of the intersecting conditions did not actually require a new function. The AutoCad circle function can accomplish the task by constructing the circle using the '3 point' option, and using what is called 'object snap tangent' for those elements that require it. However, to minimize the number of operations required to select a such a circle, we have created another shortcut routine:

> MINT Function. Median Intersect. Computes intersecting conditions exactly after the controlling elements had been determined.

The circle in Figure 10 is an example.

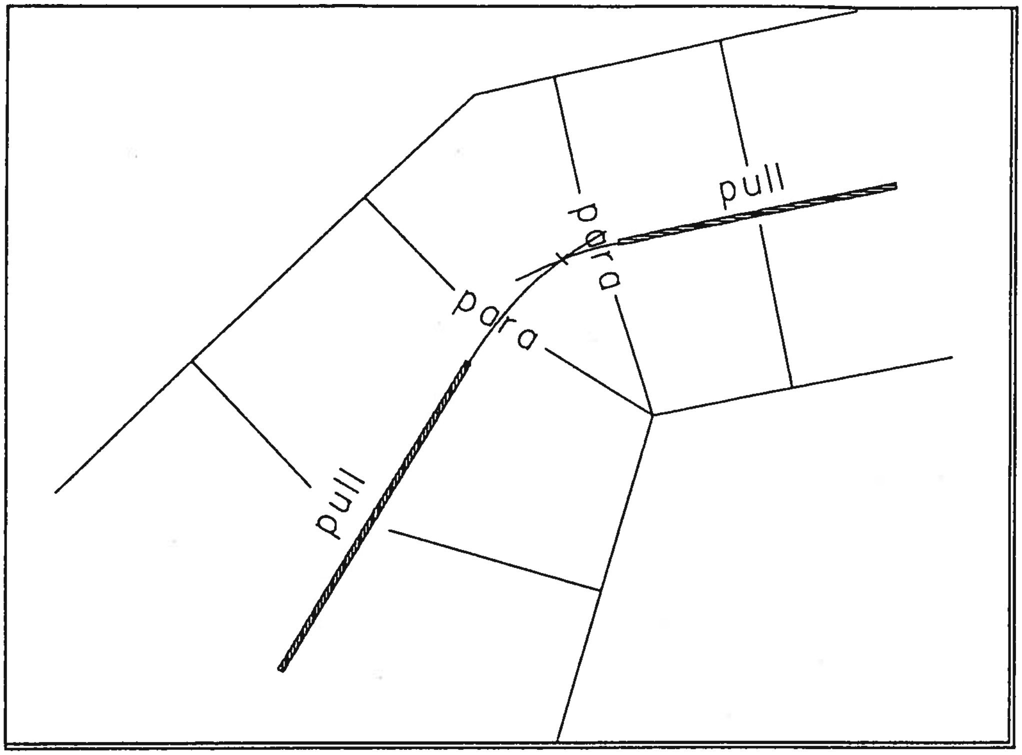

Figure 9: Using PULL and PARA Graphic Intersect

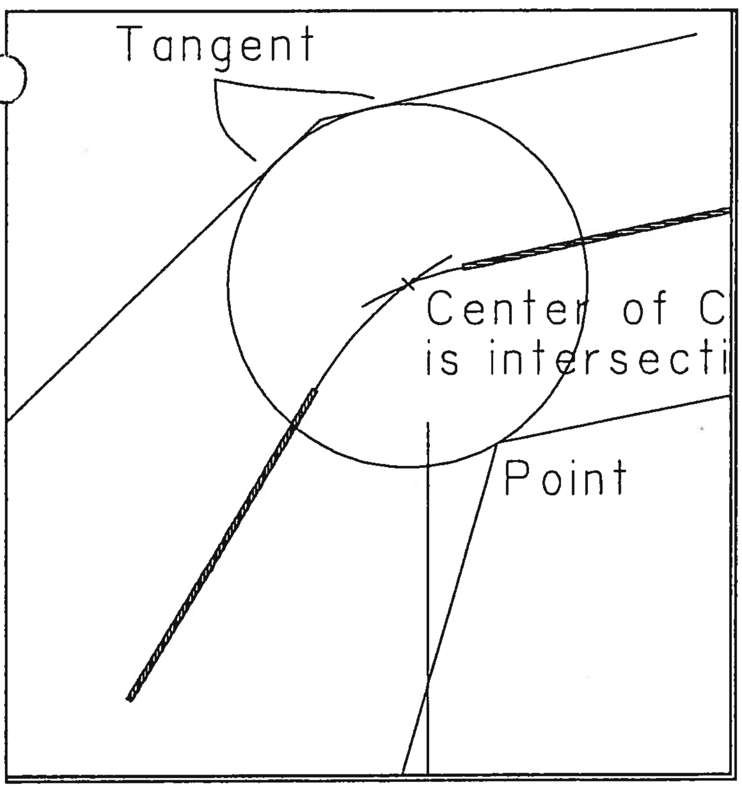

Figure 10: Use 3-Point Circle Exact Intersect

MANUAL SOLUTION WITH AUTOCAD

With the above tools, the solution involves keying in the section lines with meander corners. This can be done with coordinates or bearings and distances. Next the meander lines are entered. They can be adjusted to the meander corners, if desired. Some thought should be given to what happens to the misclosure in the section, as different procedures will result in different final meander lines.

Next, various obvious conditions can be solved, if there are any. Typically this involves spotting a few line-line conditions. Or just getting started at one end. By experimentation with moving circles it is possible to make a good guess at the next condition, then that condition can be computed, checked, etc. The process can be thought of as:

1. Compute known condition (L-L, P-L, or P-P) to its end.

2. Check trial median line section terminus to see if it is a valid point on the median line. That is, that the median line has not been affected by intersection with another condition. This can be done quickly by drawing a check circle.

3. If the end point is a valid point on the median line then the case is one of a *consecutive condition*, (no intersection). The next condition is usually obvious, and the process can begin again (with item 1.).

   or

4. If the end point is not a valid median line point, then the case is one of an *intersecting condition*. At this point you can usually figure what condition was intersected by seeing what the circle hit first. This may involve drawing circles off of prior points generated in the current condition. Once the next intersection condition is determined, the intersection can be computed, and the process proceeds accordingly, back to 1.

5. Partition lines can be dropped in perpendicular to the median line from appropriate points. It is a little difficult to precisely drop a perpendicular to the parabolic curve sections, but it can be done by using the short chord segments generated by PARA. In the future, perhaps a special exact function can be programmed for this application.

Sometimes the median line can be so complex that it is necessary to develop a *script* of the controlling conditions to keep track of it.

AUTOMATED COMPUTATION

The use of Autocad following the manual process described above makes the work of computing a median line much more efficient. However, it still requires considerable skill, knowledge and concentration in order to properly analyze and compute. On wide rivers and lakes it is often very difficult to see what the conditions will be and the solution can be a very intense guess trial and error process. The CAD approach provides tools that greatly facilitate this analysis, but human judgement is still a prime requisite, and the process can still be time consuming.

It was quickly apparent that a totally automated process was desirable, and that might be possible because the process outlined above manual steps is fairly simple, with the exception of the analysis needed to determine what the next condition is. If it is possible to formulate the rules properly, it might be possible to automate the generation of the median line totally. The process could even be useful even if it were very crude, but if the process is totally in software, it would still greatly aid in median line computations.

Such an automated system was developed by the authors in 1991 primarily through the efforts Tom Noble, an experienced AutoLISP programmer. His approach could best be described as "optimized brute force" because it does not attempt to proceed down the median line in a direct analytical process, but rather computes all combinations of conditions given all elements of each meander, then proceeds on a process of weeding out invalid conditions. Some intersections of conditions come directly out of this stage of the processing, however not all intersecting conditions are computed. After the initial weeding exercises attempts to develop a complete median line point to areas where conditions and intersections are missing. All remaining computations for intersecting conditions are then applied. After that occurs another attempt is made to construct the median line, and the process iterates until a complete median can be constructed.

The process was tested on a wide variety of real as well as manufactured rivers to assure that the program would work in most situations. There still may be geometric situations which will not solve, however the current result is a success in that it reduces a computational process that once took weeks to months to produce an approximate result, to one that may take a few hours of unattended computation.

The time needed to perform the automated computation is a function of the complexity of the bank meanders, their number, the version of AutoCAD being used, and the speed of the particular hardware being used. For example the following times are given as guidelines, and represent preliminary performance information: